Raising the Floor and Lifting The Ceiling: Math For All

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Overview

- Research overview
- Research questions
- The Classroom Milieu
- Research Findings
- Some Implications

Research Overview

- Grade 7 mathematics classroom
  - 36 students, 11 of whom had special education designations.

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Special Education Students

Students	Number of Students
Mild Cognitive Disability	1
Learning Disability	7
Gifted and Talented	2
Special School Jurisdiction Designation	1

Research Goals

- To design a study based on the principles of Universal Design for Learning;
- To determine the academic achievement of the students;
- To provide a visual image of a mathematics classroom that follows the principles of Universal Design for Learning

Research Questions

- What is the impact of Universal Design for Learning on student mathematics achievement?
- What pedagogical content knowledge is required to create mathematics classrooms in which all students succeed?

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Disability is the Mismatch between learners needs and education offered

Not a personal trait but relationship between the learner and the learning environment or education delivery

Achievement Gap

- Current schooling practices are not effective for some groups of students
- Continuing to do what we have always done will perpetuate rather than eliminate the gap
- Repeated failure over time creates an achievement gap that is exceedingly difficult to erase

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Accessibility =

- The ability of the learning environment to adjust to the needs of all learners (aka. personalization)
- Flexibility of the educational environment, curriculum and delivery
- Availability of adequate alternative-but-equivalent learning content and activities
- A learning design that accommodates the same learning objective/outcome but diverse activities and paths

Universal Design for Learning

UDL offers broader access to the curriculum by all students by:

- Providing Flexible Means of Representation (Input)
- Providing Flexible Means of Expression (Output)
- Providing Flexible Means of Engagement (Interest)

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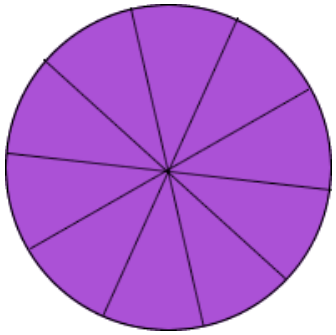
Instruction: STUDENT TASK

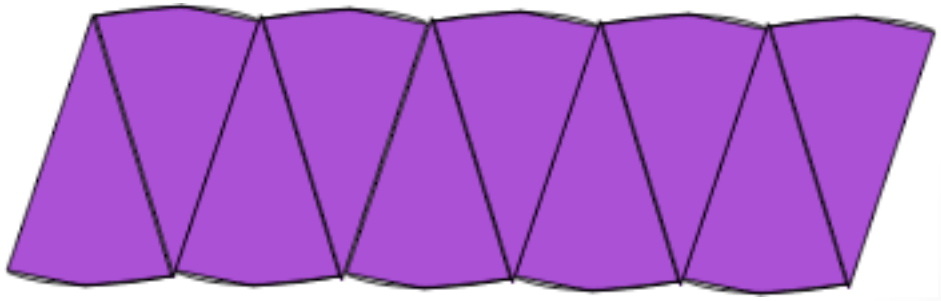
- **Lines, Shapes and Spaces – Task**
- Everywhere we see lines, shapes and spaces. Working with a partner, you will need to:
  - Find lines, shapes and spaces in the classroom, school or outside of the school.
  - Take digital pictures of a variety of lines, shapes and spaces you find.
  - Download these pictures onto your laptop.
  - Sort through your pictures and decide on at least two from each category, 1-D, 2-D and 3-D that you want to examine in great detail.
  - Now, drag each of the pictures into a Word™ document. (Make sure you save each one with its own name and in a place you can both access.)
- Working with your partner, name, describe, analyze and measure the various images you have collected.
- Select a way to present your work that best expresses how you understand the ideas in this task.

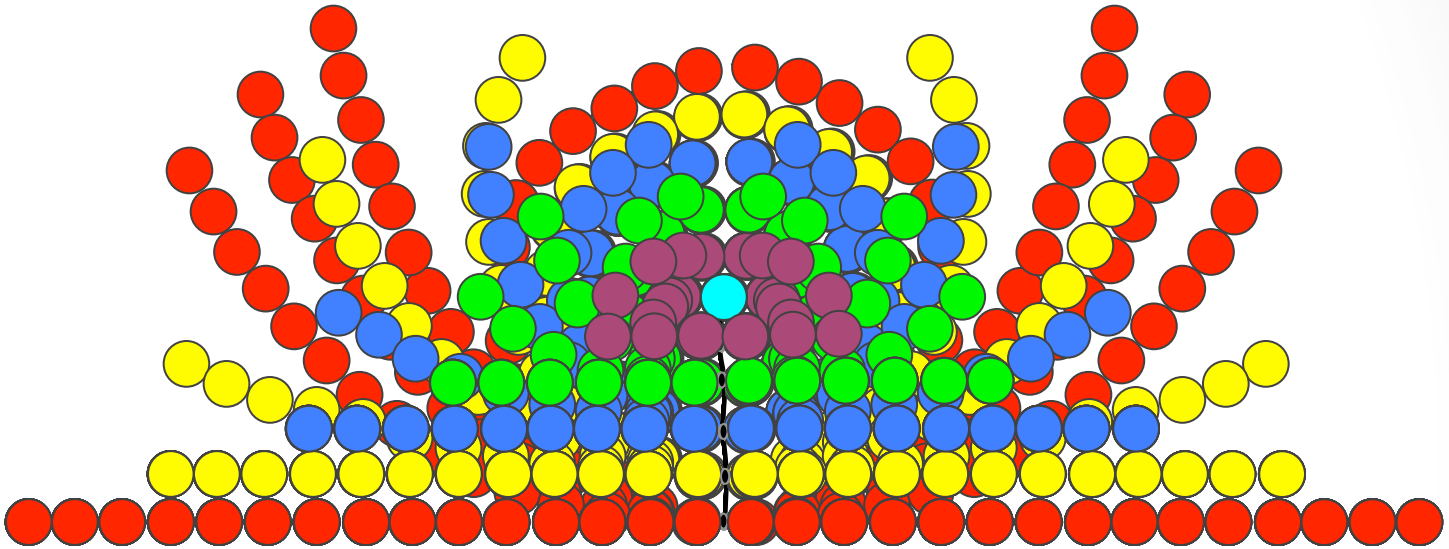
Instruction: STUDENT TASK

Pi are squared

- The circumference of a circle of radius *r* is $2\pi r$. Suppose you have a round pie, and 1000 of your best friends come over for dessert. You divide the circular pie into 1000 equal-sized pieces by cutting from the centre. Then, before serving them, you rearrange the pieces by putting the first piece with the curved part up, the next piece right next to it with curved edge down, the next one with curved side up, alternating until all 1000 pieces are arranged. The shape you have constructed is almost and exact rectangle except that its top and bottom edges are made of 500 just slightly curved tiny segments that came from the edge of the pie. How long is the rectangle? How wide is it? What is its area? Why is this story a convincing demonstration that the formula for the area of a circle is correct?







Imagine a circle made out of strands of beads.

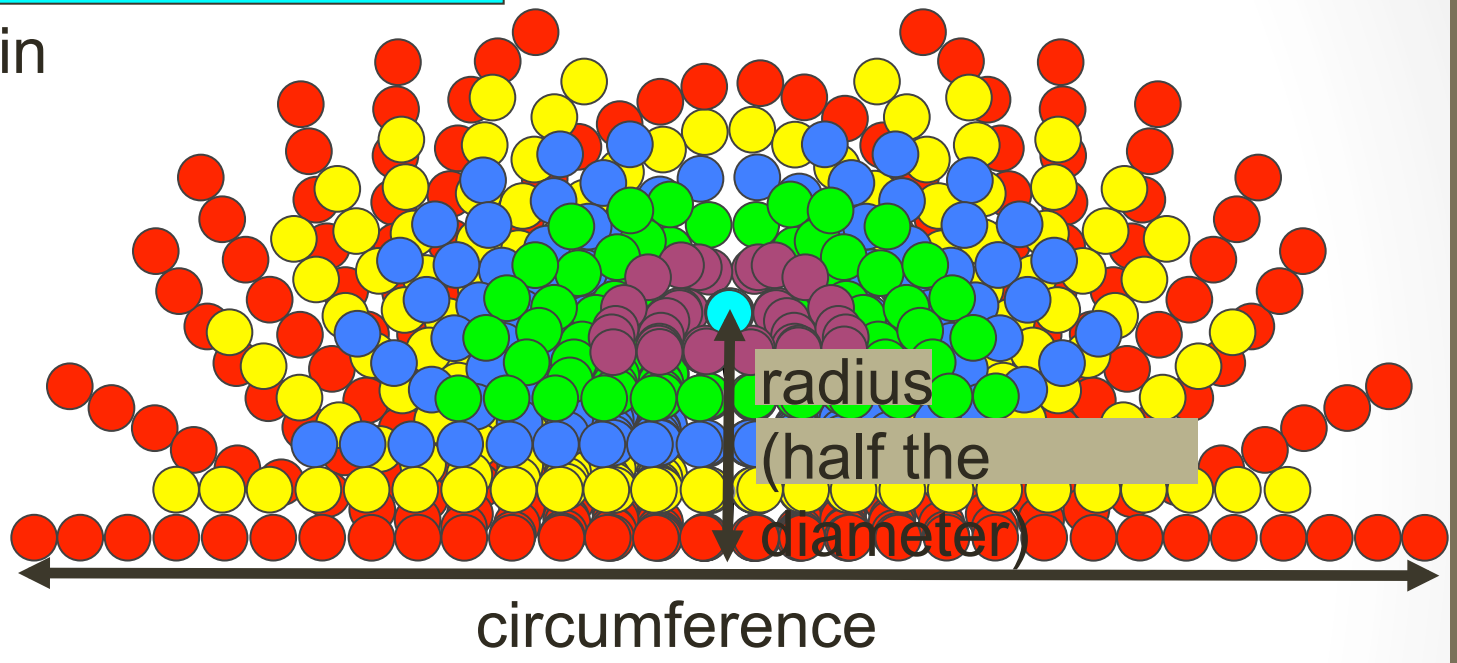
Open it out.

Click to see the circle open

Let's watch that again.

Click to see the circle

open again

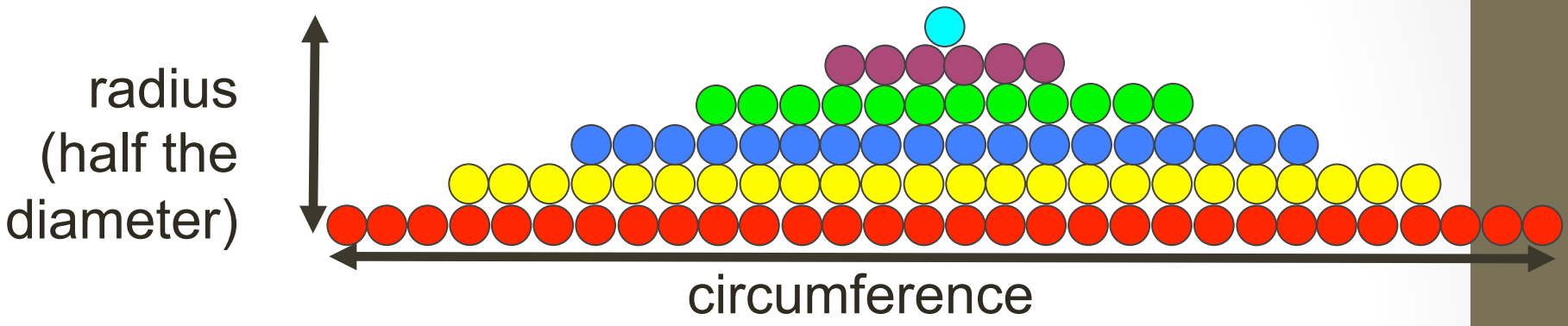


It's a **triangle**!

base = circumference

height = radius
(half the diameter)

We know how to find the area of a triangle.



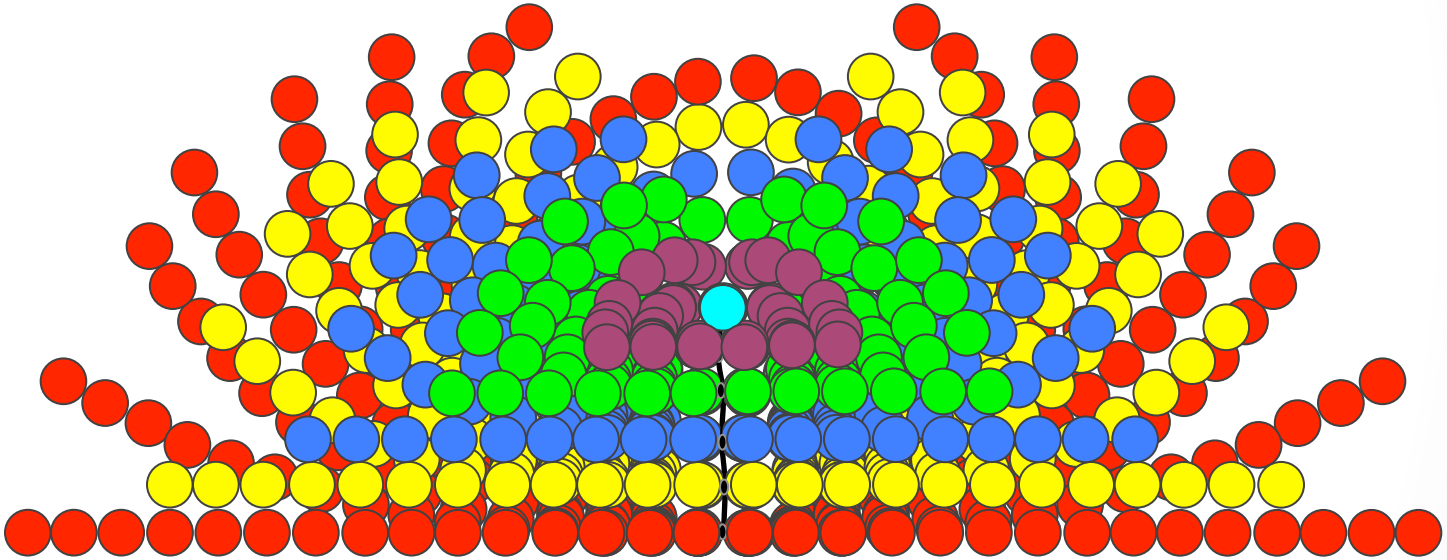
circle

Area of the ~~triangle~~ = $\frac{\text{Base} \times \text{Height}}{2}$

= $\frac{\text{Circumference} \times \text{Radius}}{2}$

Summary

$$\text{Area} = \frac{\text{Circumference} \times \text{Radius}}{2}$$



Instruction: STUDENT TASK

FOLLOWING THE MOON

- In the evening or night sky, take a digital picture of the moon. Using a compass find the direction of the moon. Take another picture at a different time and find the direction. Has the moon moved positions? Does the moon appear different? Why or why not?

- Use a graphing program for the next few steps.

- First plot the sun in the centre of the x and y axis. How far is the sun from the earth? Make this distance the radius of the earth's orbit around the earth. Assume the orbit is circular.
  - Draw the orbit of the earth around the sun. Now plot the earth on that orbit. Make sure the plot of the earth is accurate according to the current season. Hint: In our northern winter, the earth's north pole points away from the sun. Which direction does the earth travel (clockwise or counter clockwise)? How do you know?

- Where would the earth be for the solstices and the equinoxes? Which direction does the earth travel (clockwise or counter clockwise)? How do you know? Now plot the earth according to its relative position given the season and the number of days travelled.

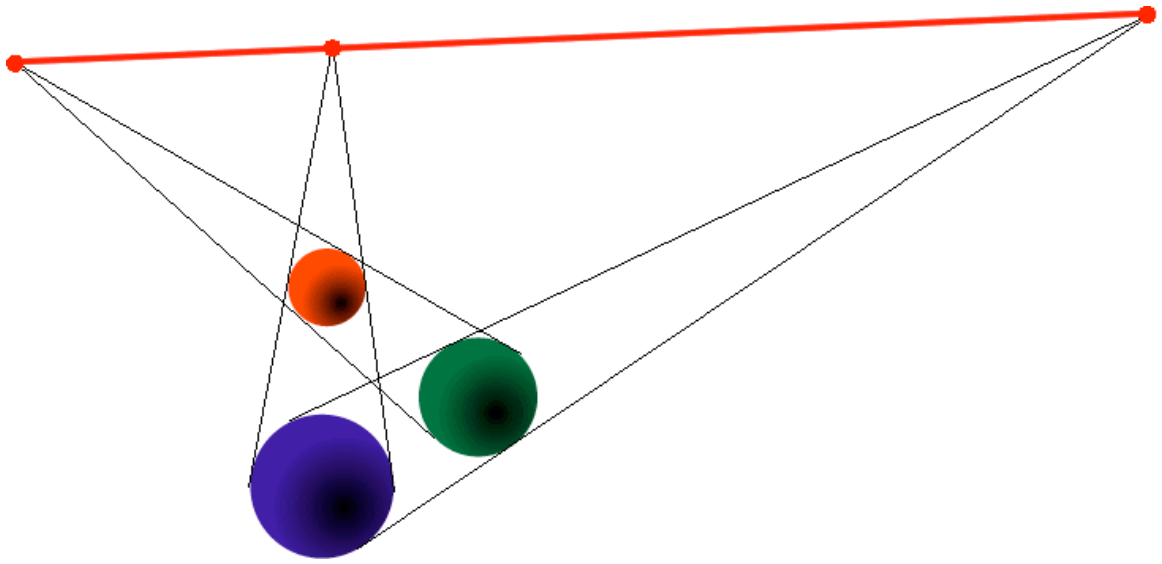
  - Draw the orbit of the moon around the earth. Plot the moon on the orbit according to the phase the moon displays in your picture. Where would the new moon be on this orbit? Where is the full moon on the orbit? Where is the quarter moon? Is there more that one location for the quarter moon on the orbit?

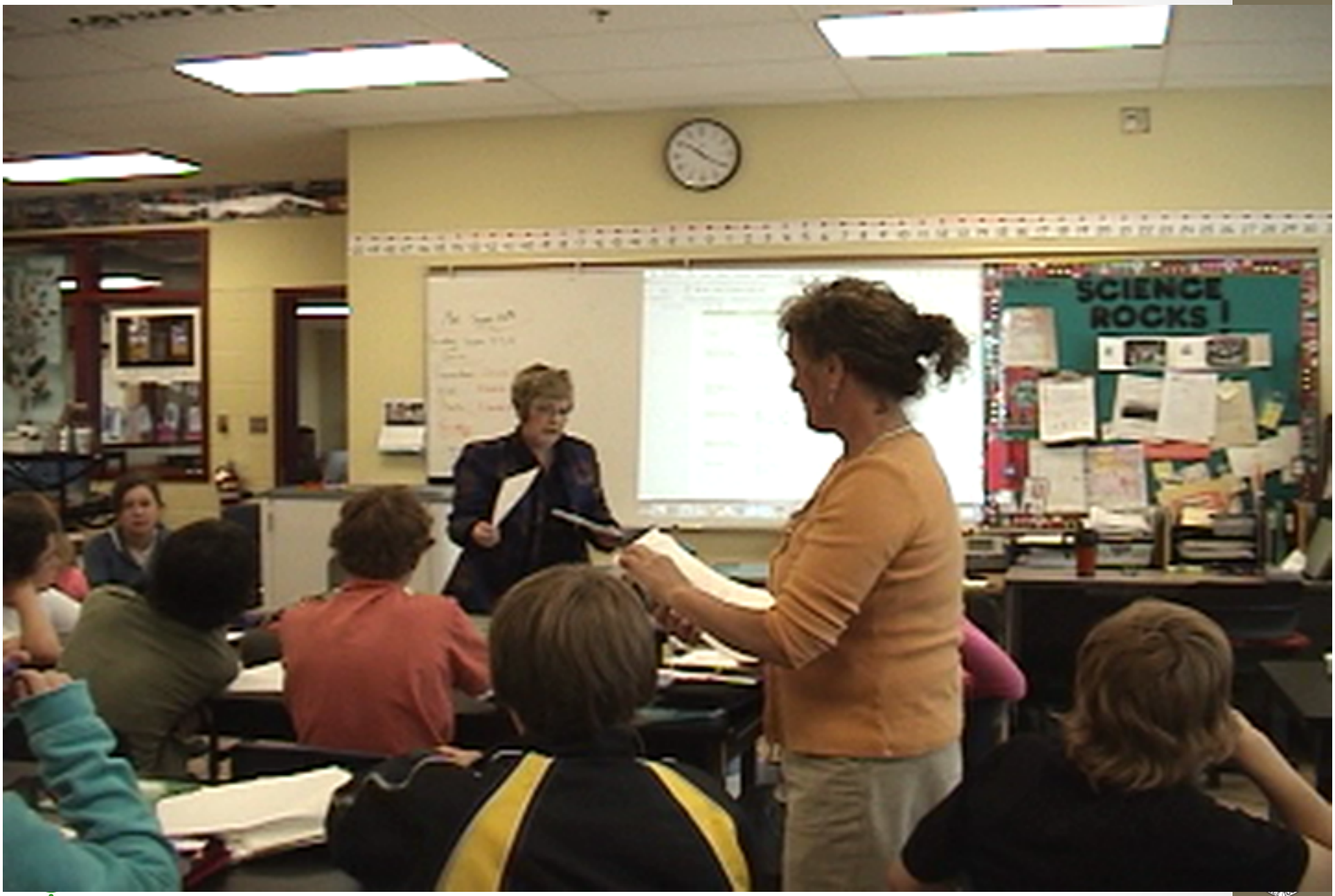
- Next, draw lines to connect the center points of the sun, moon and earth. You should now have a triangle. What are the angle measurements of each? What are the angles for the new moon, full moon and quarter moon(s)?

Instruction: STUDENT TASK

Tangents to the Circle

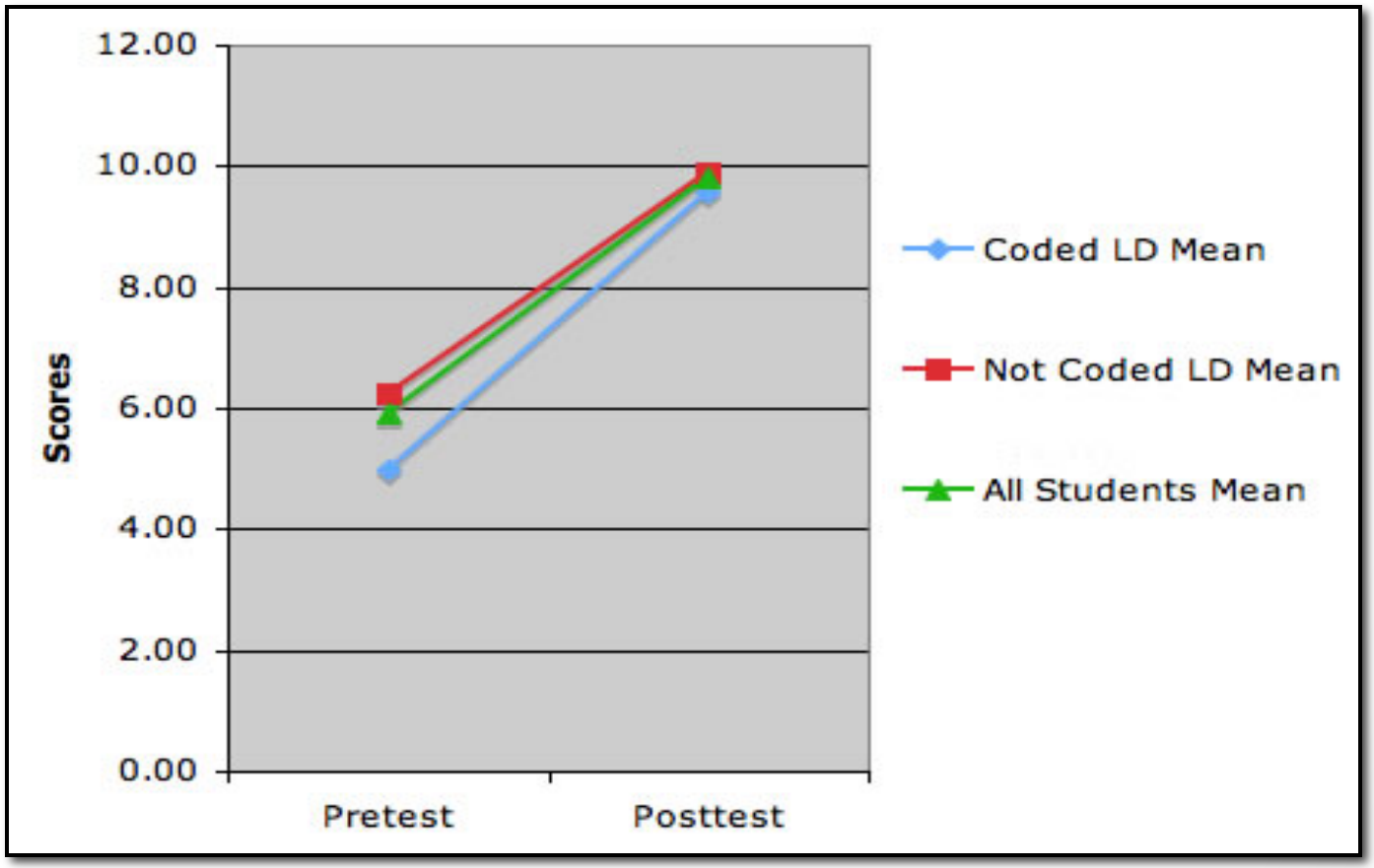
- Three circles of different sizes are distributed randomly, as shown. Pairs of tangents are drawn around the circles, with a surprising result: the three intersection points for the tangents lie along a straight line.
- Is this a coincidence, or will it always happen?



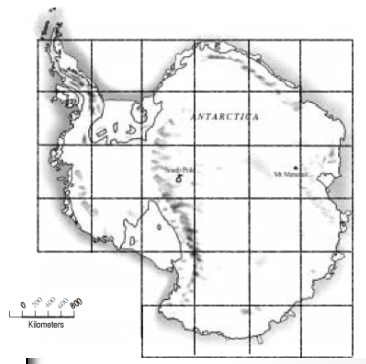

SCIENCE
ROCKS!

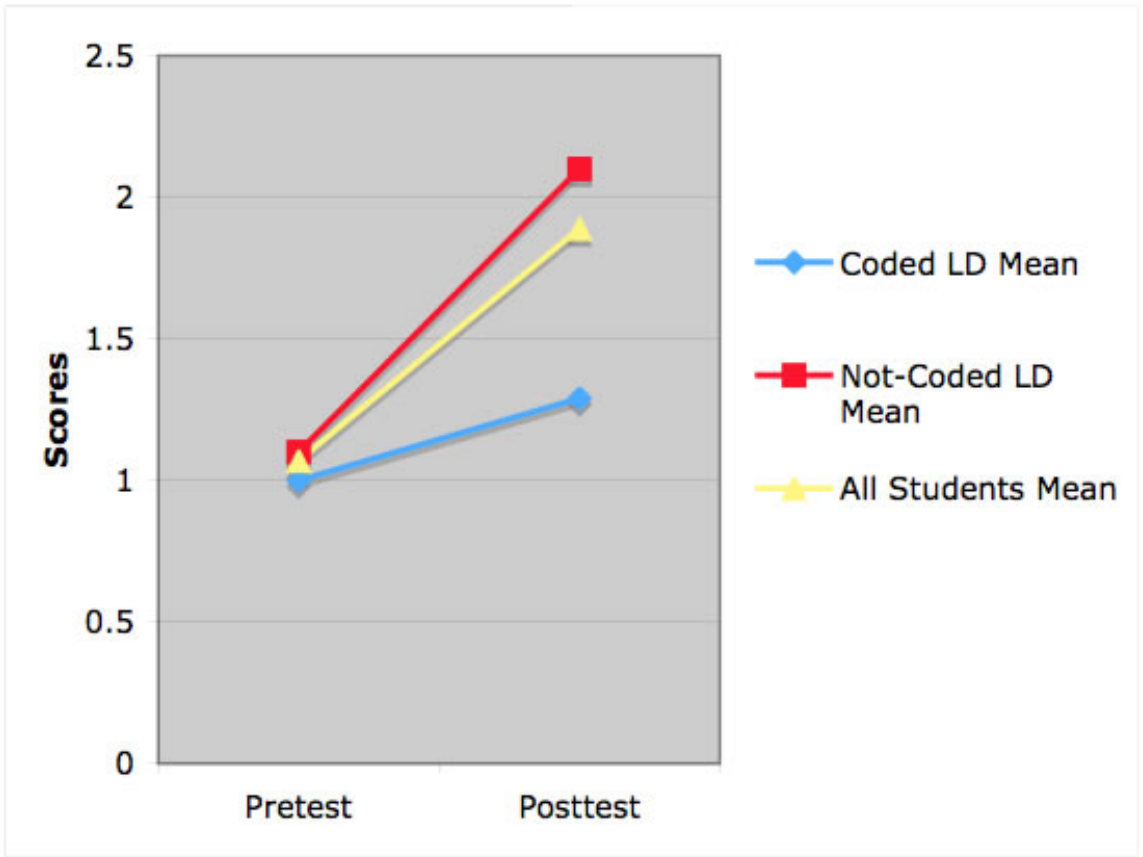
Findings

1. All students showed significant improvement in achievement.



Task 1: Continent Task





Scaled Area Of An Irregular Shape

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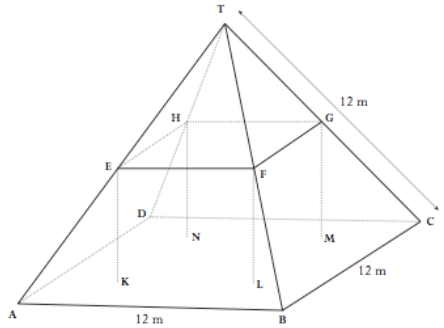
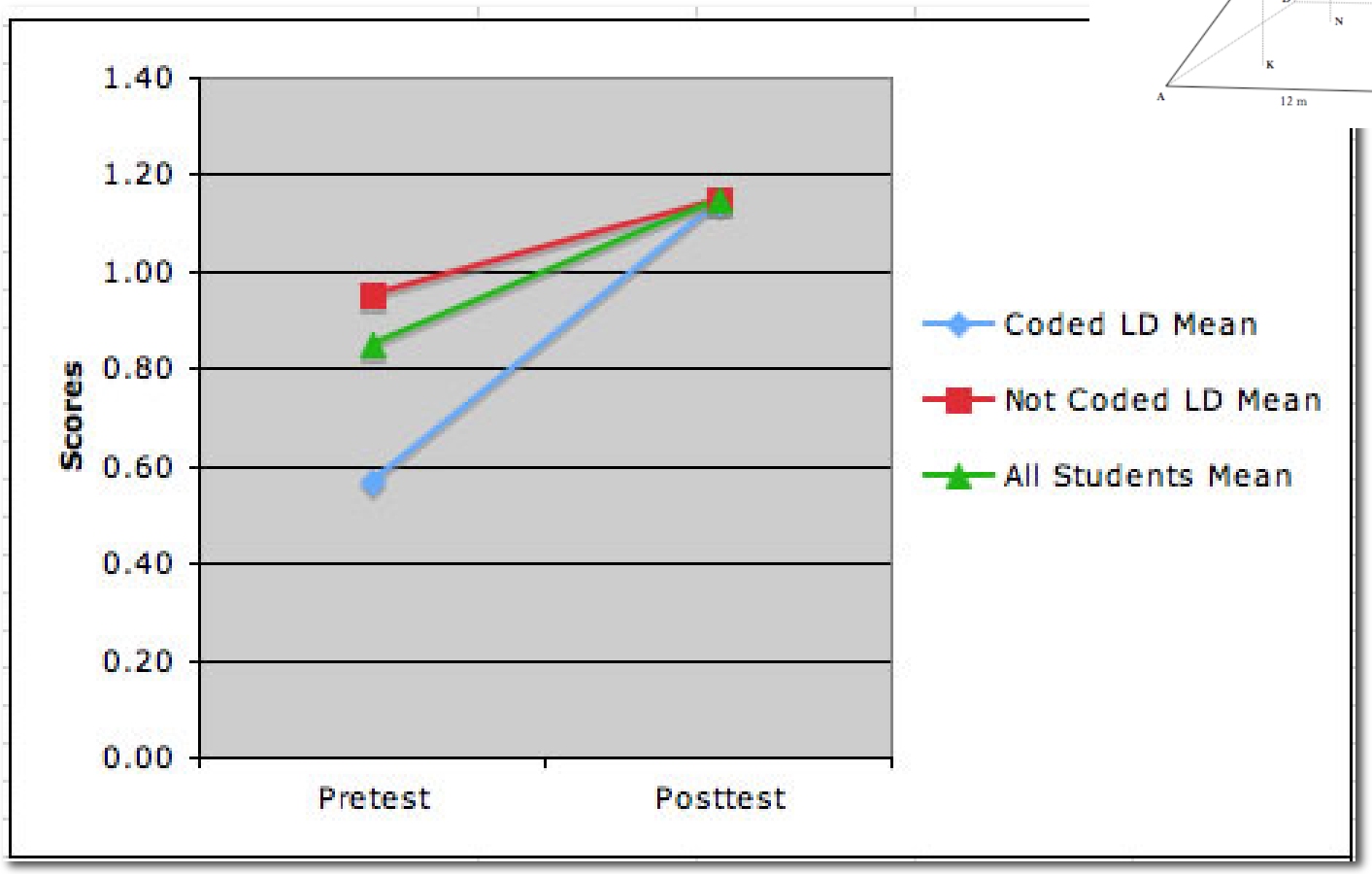
Task 2: Farm Task





Perimeter and Area

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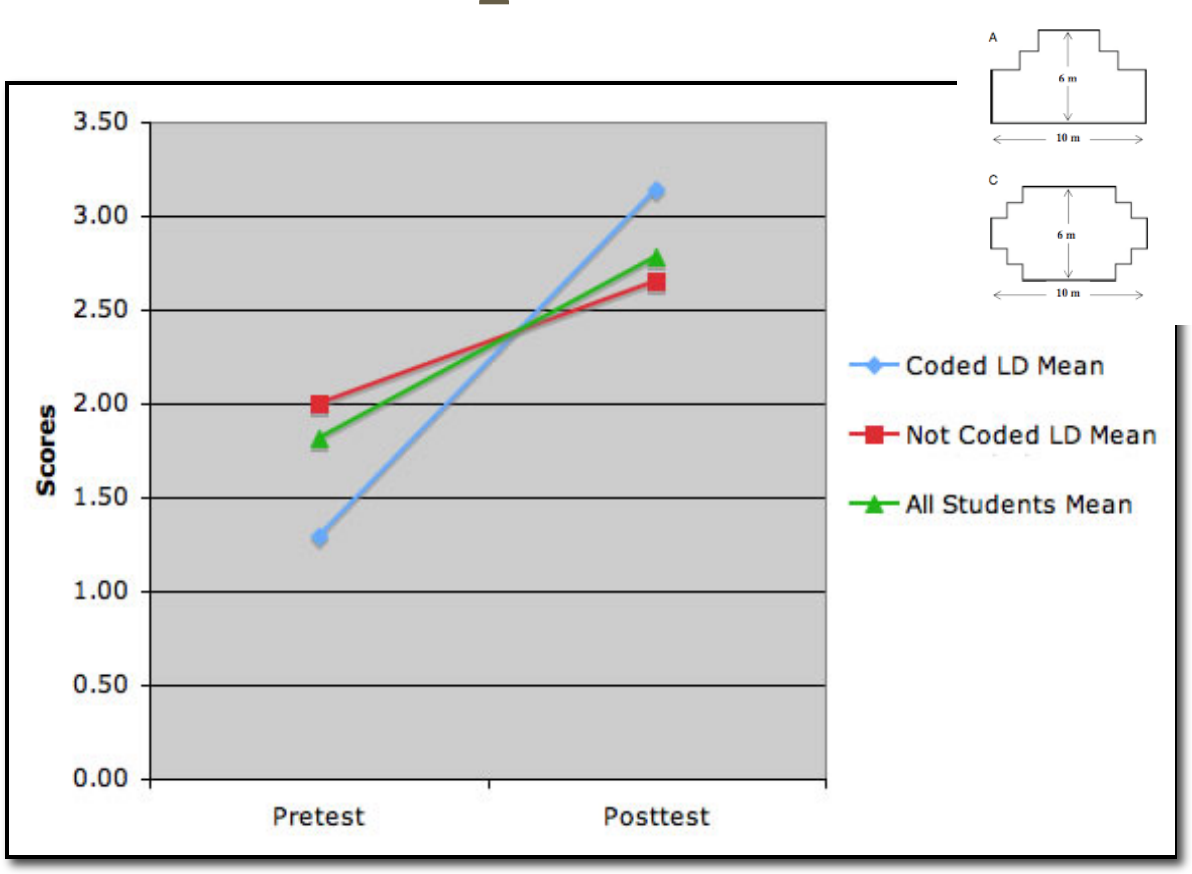
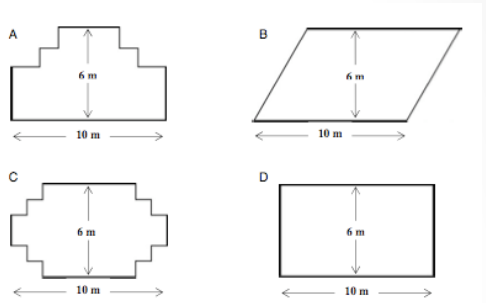
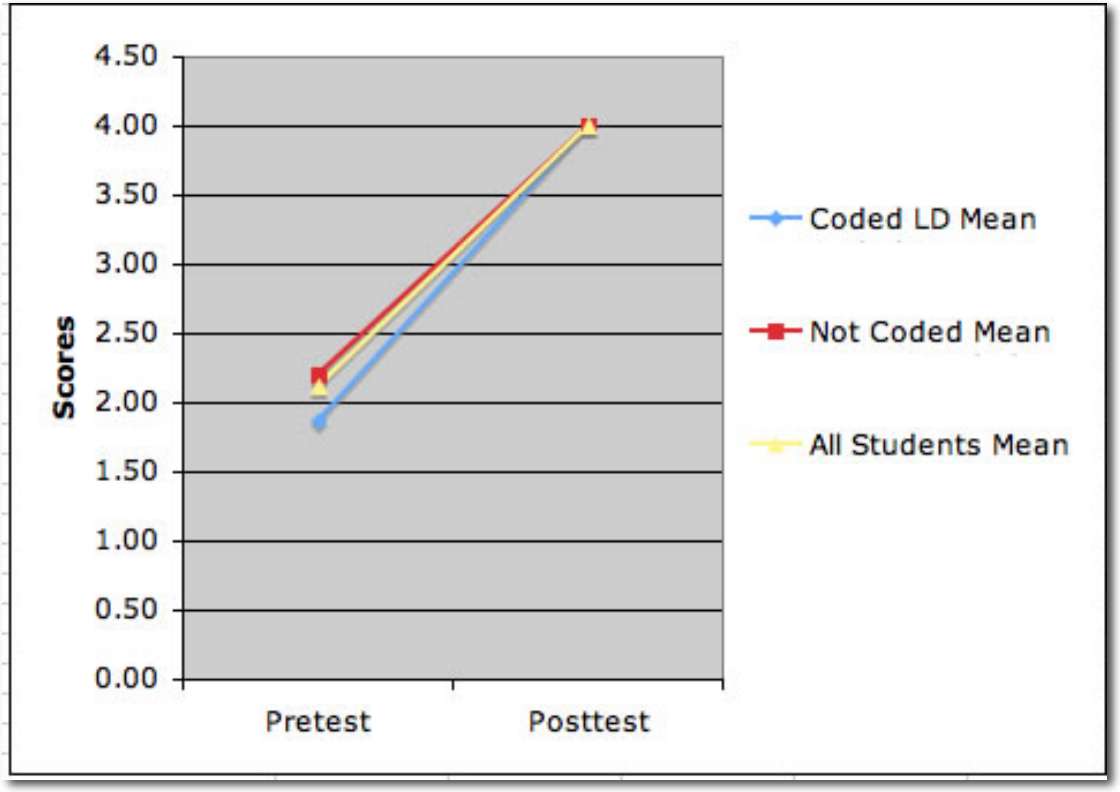
Task 3: Carpenter Task





Calculate perimeters for compound Irregular shapes

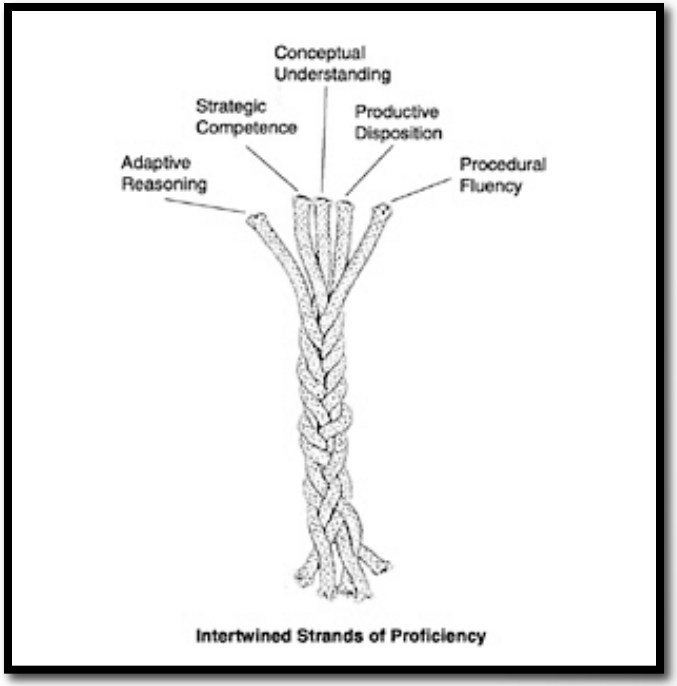
Task 4: Twisted Building



Calculate calculate degrees of rotation and determine orientation following a number of turns

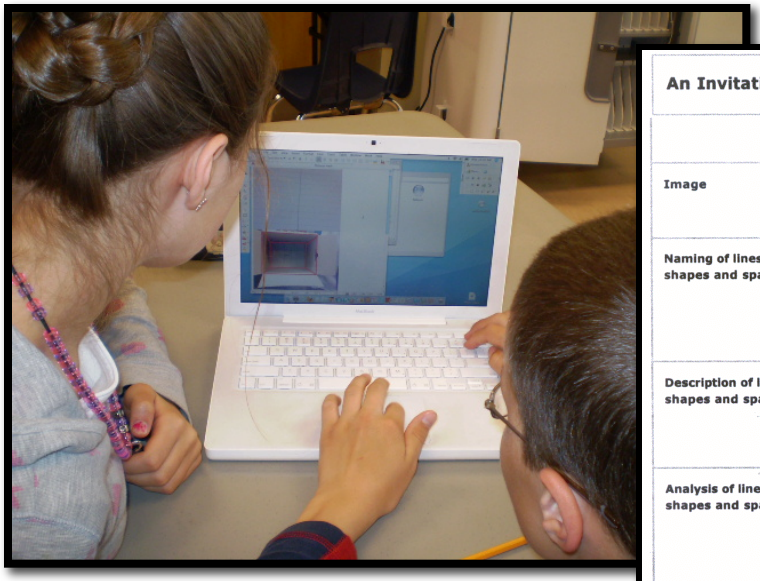
Findings (cont.)

2. All students demonstrated gains in the five strands of mathematical proficiency.
   - Conceptual understanding
   - Procedural fluency
   - Strategic competence
   - Adaptive reasoning
   - Productive disposition



Findings (cont.)

3. All students can engage with difficult mathematical ideas when they are provided with dynamic assessment.



An Invitation: Lines, Shapes and Spaces

	Keep Working	Getting There	You've Got It	In The Flow!
Image	Digital photo is blurred or not clear.	Digital photo is clear however the image is vague.	Digital photo is clear with an identifiable image.	Digital photo is clear with a sharply focused image.
Naming of lines, shapes and spaces.	Using own naming structure.	Names lines, shapes and spaces using a mix of standard mathematical conventions and improvised names.	Accurately names lines, shapes and spaces using standard mathematical conventions.	Accurately names lines, shapes and spaces and provides details about which "family" the line, shape or space is part of.
Description of lines, shapes and spaces.	Provides a description using own words.	Provides a description that uses a mix of mathematical terminology and own improvised description.	Provides an accurate description using mathematical terminology.	Provides a detailed description that helps to illuminate features of the line, shape and space.
Analysis of lines, shapes and spaces.	Unable to figure out the various lines, shapes and spaces that compose this figure.	Discerns some of the properties that comprise this figure.	Accurately discerns most of the properties that comprise this figure.	Analysis of this figure helps to illuminate the properties showing how this lines, shape or space relates to other figures.
Measures lines, shapes and spaces	Finds a way to measure some of the lines, shapes and spaces.	Finds different ways to measure the lines, shapes and spaces.	Accurately measures the figure in a variety of different ways.	Accurately measures the figure using a variety of ways and measuring tools.
Teamwork	Conflicts between team members interferes with work quality and production.	Cooperative team work in which team members reinforce each other's learning.	Effective team work in which team members build on and extend each other's ideas.	Effective team work in which team members build on, extend and provide feedback each other's ideas.

Comments:

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Findings (cont)

4. The principles of UDL permit teachers to break the stranglehold of the procedural script for teaching mathematics.
5. Access to technology is a critical factor.
6. Introducing UDL is a disruptive innovation – disrupting a fixed mindset.
7. Creating accessible mathematics classrooms consistent with UDL principles requires increased teacher knowledge and support for ongoing professional development.

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Issues for Teachers

- Weak understanding of mathematics for teaching
- Weak understanding of how people learn
- Limited use of technologies
- Limited understanding of the power of model creation to build variation of concept presentation and use
- An instructional script that emphasizes rehearsal of memorized procedures

Implications

- Mathematics for Teaching
- Task Design
- Formative Assessment
- Appropriate Role for Learning with Technology