Information and Markets

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Summer School – PIMS/UBC

July 2006

Lecture 2: Mechanism Design

Akerlof: only low quality automobiles are traded in market

Is there *some* mechanism that does better?

Akerlof as REE

- M sellers, $N - M > M$ buyers
- M goods
- states Ω = vectors of qualities = $\{1, 4\}^M$
- signals $S_i = \{1, 4\}$
- REE price function $p : S \to \mathbb{R}^M$

$$p_i(s) = s_i + 1$$

This seems silly: why would sellers with low quality automobiles – who are *only* agents who know the quality of their own automobile – divulge that quality which can only harm them?

Differently: this REE is not incentive compatible.

$\longrightarrow$ Look for incentive compatible mechanism

Revelation Principle Any outcome that can be achieved by *any* incentive compatible mechanism (i.e. achieved as the outcome of a Bayesian Nash game) can be achieved by a direct mechanism subject to participation (individual rationality) constraints and incentive compatibility constraints.

Direct Mechanism Agents send messages about types, outcome implemented as a function of messages.

Constraints: if others behave truthfully then

- IR: agents willing to participate in mechanism:

- IC: agents willing to send truthful messages

Mechanism for this problem

- buyers passive
- seller message = type: $L = 1, H = 4$
- $\pi =$ probability of sale, $t =$ transfer to seller

  (NOT price contingent on sale)

$$
\begin{aligned}
\text{IRB:} \quad \tfrac{1}{2}[\pi(L)2 - t(L)] + \tfrac{1}{2}[\pi(H)5 - t(L)] &\geq 0 \\
\text{IRH:} \quad t(H) - \pi(H)4 &\geq 0 \\
\text{IRL:} \quad t(L) - \pi(L)1 &\geq 0 \\
\text{ICH:} \quad t(H) - \pi(H)4 &\geq t(L) - \pi(L)4 \\
\text{ICL:} \quad t(L) - \pi(L)1 &\geq t(H) - \pi(H)1
\end{aligned}
$$

Can efficient outcome be achieved? No:

- $\pi(H) = \pi(L) = 1$
- ICH $\Rightarrow t(H) \geq t(L)$
- ICL $\Rightarrow t(L) \geq t(H)$
- IRH $\Rightarrow t(H) \geq 4$
- $t(L) = t(H) \geq 4$ violates IRB

Can we do better than Akerlof market solution?

Yes $\pi(L) = 1$, $t(L) = 1.5$, $\pi(H) = 0.1$, $t(H) = .45$

Exercise Find a mechanism that maximizes

$$\text{social gain} = \frac{1}{2}\pi(H)(1) + \frac{1}{2}\pi(L)1$$

The analysis above tacitly assumes

- all sellers make same report
- all sellers treated same

For efficiency this does not matter

- number the sellers $1, 2, \ldots M$
- vector of reports $r = (r_1, \ldots, r_M)$
- $\pi_i(r_i, r_{-i}), t_i(r_i, r_{-i})$
- $\pi_i(r_i) = E\pi_i(r_i, r_{-i})$, $t_i(r_i) = Et_i(r_i, r_{-i})$
- same inequalities for i

$\Rightarrow$ same bound on social gain for i's automobile

$\Rightarrow$ same bound on per-capita social gain

Exercise Find a mechanism that maximizes

$$\text{social gain} = \frac{1}{2}\pi(H)(1) + \frac{1}{2}\pi(L)1$$

Exercise What happens if agents are risk averse?

McLean-Postlewaite

- all cars same quality
- sellers know quality
- buyers know only distribution of quality
- distribution of quality uniform on $1, 4$
- #sellers ≥ 3

Efficient mechanism

- sellers report quality
- transfer automobiles at prices that depend on *all* reports
  - if (# H reports) $\geq \frac{1}{2}$ (#sellers): $t(H) = 4.5, t(L) = 4.4$
  - if (# H reports) $< \frac{1}{2}$ (#sellers): $t(H) = 1.4, t(L) = 1.5$

IRB, IRH, IRL, ICH, ICL ?

- everyone always makes strict gain
- no one ever gains by misrepresenting

Seller information imperfect?

Assume

- sellers receive signal of true quality

	H	L
H	ρ	$1-\rho$
L	$1-\rho$	ρ

- $\rho > .5$ (signal is informative)
- signals independent conditional on true quality

For $\rho > .5$ same mechanism works if M large enough

- M large $\Rightarrow$ majority is nearly perfect predictor

- if misrepresentation does not change majority

  - misrepresentation gains $+.1$ or loses $-.1$

  - misrepresentation loses more often than gains

- if misrepresentation changes majority

  - may gain lot

  - unlikely

Another variant

$$u_s(H,m) = 4 + m \quad u_s(L,m) = 1 + m$$
$$u_b(H,m) = 5 + m \quad u_s(L,m) = 0 + m$$

Modification majority report = L $\rightarrow$

- do *not* transfer automobile

- make monetary transfers

Mechanism is *almost* efficient if #sellers large

Difference between Akerlof and McLean-Postlewaite environments?

- Akerlof: state = vector of qualities

- misreport *certain* to change perceived state

- McLean-Postlewaite: state = true quality

- misreport *unlikely* to change perceived state

- McLean-Postlewaite: agents are informationally small

McLean-Postelwaite

- formal definition: informationally small
- prove: agents are informationally small $\Rightarrow$ almost full information revelation is incentive-compatible