Information and Markets

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Summer School – PIMS/UBC

July 2006

Lecture 1: Rational Expectations

Classical models

- all agents equally informed
- all agents have equally good analysis
- nothing to be learned from others

World

- some agents have inside information
- some agents have superior analysis
- much to be learned from others

Especially true in financial markets

- insiders
- analysts, advice from brokers

Questions

- do informational asymmetries matter?
- is information revealed in prices?

  (efficient market hypothesis)
- is information revealed in market behavior?
- what models are appropriate/useful?
- what is the historical evidence?
- what is the experimental evidence?

Three approaches

- general equilibrium theory
- mechanism design
- auctions

Akerlof

- many buyers, many sellers
- buyers each endowed with money
- sellers each endowed with one automobile
- automobile quality q
- $u_s(q,m) = q + m$, $u_b(q,m) = 1 + q + m$
- sellers know quality of own car & distribution of quality
- buyers know only distribution of quality
- distribution of quality uniform on $1, 4$

Market equilibrium?

- all automobiles must have same price p
- $p \geq 4$ $\Rightarrow$ average value = 3.5 $\Rightarrow$ no buyers willing to buy
- $p < 4$ $\Rightarrow$ no high quality automobiles sold
- $\Rightarrow$ $1 \leq p \leq 2$, only low quality automobiles sold

More dramatic version of Akerlof

- $u_s(q, m) = q + m$, $u_b(q, m) = 1.5q + m$
- quality uniformly distributed on $[0, 1]$

Market equilibrium?

- all automobiles must have same price p
- $p \geq 1 \Rightarrow$ average value = .75 $\Rightarrow$ no buyers willing to buy
- $p < 1 \Rightarrow$ average quality sold = $.5p \Rightarrow$ average value = $.75p$
- $\Rightarrow p = 0$, no automobiles sold

General equilibrium model

- N agents
- states of the world Ω
- common priors Prob on Ω
- signals $S = S_1 \times \ldots \times S_N$
- joint probabilities Prob on $\Omega \times S$
- commodity space $\mathbb{R}^J$
- endowments $e_i \in \mathbb{R}^J$
- utilities $u_i(x; s_i, \omega)$

Rational expectations equilibrium REE

- price function $p : S \to \mathbb{R}^J$
- choices x_i
- agents optimize subject to budget constraint, given own signals and information in prices
- markets clear

Interpretations

- x = ordinary goods, state = weather, utilities depend on weather, some agents read weather forecasts

- x = goods of varying quality, utilities depend on quality, some agents know more about quality

- x = assets, (expected) utility depends on asset payoffs, some agents know more about true distribution of asset payoffs

Example

- two agents A, B
- two states H, T
- A perfectly informed

  $s_A = H, T$ perfectly correlated with true state
- B perfectly uninformed

  $s_B = H$ independent of true state
- $\text{Prob}(H) = \text{Prob}(T) = .5$

- two goods x, y
- endowments $e_A = e_B = (1,1)$
- utilities

$$\begin{aligned} u_A(x,y;H) &= \frac{2}{3}\log x + \frac{1}{3}\log y \\ u_A(x,y;T) &= \frac{1}{3}\log x + \frac{2}{3}\log y \\ u_B(x,y;H) &= \frac{2}{3}\log x + \frac{1}{3}\log y \\ u_A(x,y;T) &= \frac{1}{3}\log x + \frac{2}{3}\log y \end{aligned}$$

REE?

- $p_x + p_y = 1$; write $q = p_x$
- $p_x + p_y = 1 \Rightarrow$ wealth $= 2$
- in each state: demand $=$ supply, then solve for q

- $q(H) \neq q(T) \Rightarrow$ then both agents know state $\Rightarrow$

$$\text{state} = H: \quad \frac{2}{3}\frac{1}{q(H)} + \frac{1}{3}\frac{1}{q(H)} = 2$$

$$\text{state} = T: \quad \frac{1}{3}\frac{1}{q(T)} + \frac{2}{3}\frac{1}{q(T)} = 2$$

$\Rightarrow q(H) = q(T)$

- $q(H) = q(T) \Rightarrow A$ knows state, B does not $\Rightarrow$

$$\text{state} = H: \quad \frac{2}{3}\frac{1}{q(H)} + \frac{1}{2}\frac{1}{q(H)} = 2$$

$$\text{state} = T: \quad \frac{1}{3}\frac{1}{q(T)} + \frac{1}{2}\frac{1}{q(T)} = 2$$

$\Rightarrow q(H) \neq q(T)$

$\Rightarrow$ no REE exists

Lessons

- REE requires individuals to know great deal about economy
- REE may not exist
- S finite $\Rightarrow$ "generically" there exists REE with p one-to-one

  fully revealing REE
- if REE is fully revealing, information is worthless
- If REE is fully revealing, and information acquisition is costly,

  why should any agent pay to acquire information?

Grossman-Stiglitz

- costly information acquisition
- noise $\longrightarrow$ imperfectly revealing REE
- limit: as cost of information, noise $\rightarrow 0$?

Two assets

- bond: payoff $= 1$

- risky asset: payoff $= \theta + \varepsilon$

  θ observable at cost

  ε unobservable

Two kinds of agents

- rational traders: mass 1

  optimize according to their information, inferences

- irrational/noise traders: mass n

  sell x units each (independent of price, information)

Cost of information (learn θ) $= c$

Suppose fraction λ of rational traders becomes informed

At equilibrium, demand = supply:

$$\lambda D_I(p|\theta) + (1-\lambda) D_U(p|\text{inference about } \theta) = nx$$

REE is price functional p^*_λ (function of θ, x) such that when uninformed traders update correctly we have

$$\lambda D_I(p^*_\lambda(\theta,x)|\theta) + (1-\lambda) D_U(p^*_\lambda(\theta,x)|p^*_\lambda) = nx$$

If

- utilities are exponential
- all variables jointly normally distributed

then

- REE $p^*_\lambda(\theta, x)$ exists and is well-behaved

Endogenize λ?

- define λ^* by relation that informed agents, uninformed agents get same expected utility:

$$Eu(1 - c - pD_I, D_I) = Eu(1 - pD_U, D_U)$$

- utility of informed agents decreases as λ increases

- $\Rightarrow \lambda^*(c, Var(x))$ well-defined

$$\lim_{c \to 0, Var(x) \to 0} \lambda^*(c, Var(x)) = \ ?$$

limit between 0, 1 $\rightarrow$ Radner model

But

- $$\lim_{c \to 0} \lim_{Var(x) \to 0} \lambda^*(c, Var(x)) = 0$$

- $$\lim_{Var(x) \to 0} \lim_{c \to 0} \lambda^*(c, Var(x)) = 1$$