Vertical Control of Inventory and Pricing Decisions

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Abstract

This paper offers a simple approach to the theory of decentralizing inventory and pricing decisions within a distribution system. We consider an upstream manufacturer selling to two outlets, which compete as differentiated duopolists and face uncertain demand. Demand spillovers between the outlets arise in the event of stock-outs. The price mechanism, in which each outlet simply pays a wholesale price and chooses price and inventory, never coordinates incentives efficiently. Contracts that can elicit first-best decisions include resale price *floors* or buy-back policies (retailer-held options to sell inventory back to the manufacturers) with fixed fees. The combination of a buy-back option plus a resale price *ceiling* elicits the first-best without the need for a fixed fee and is robust to asymmetry in information about demand at the time of contracting.

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1 Introduction

The choice of optimal inventory for a firm facing uncertain demand is a classic problem of economics (Arrow et al (1951)) and has become a central problem of management science. A recent literature recognizes that the choice of optimal inventory is not just a single agent decision problem but rather involves the alignment of incentives all along a vertical supply chain. And as Deneckere, Marvel and Peck (1996, 1997) have shown, the vertical control of inventory decisions is tightly linked to control of pricing.

This paper introduces a simple framework that synthesizes and extends the theory of decentralizing inventory and pricing decisions. Can a manufacturer simply set a wholesale price for its product, relying on the distributors of its product to set optimal pricing and inventory levels? If not, which contracts elicit the right incentives for these decisions? Our framework adopts two principles. First, an organization faces an incentive problem when an agent within the organization does not appropriate the full collective benefits of her actions. Second, an incentive problem is resolved when some of the agent's actions are constrained at the optimal levels and prices or reward systems internalize the externalities imposed by the remaining actions on other agents within the organization.

We apply this framework to analyse the roles of a range of instruments. **Resale price maintenance** in the form of vertical price floors was very common when it was legal.[1] The opposite restraint, a **vertical price ceiling**, is also observed.[2] A **buyback policy** refers to a put option to sell unused inventory back to the manufacturer or an agreement to give credit for any unsold inventory. Bookstores, for example, send the covers of paperback books and magazines to publishers to obtain credit. **Royalties** are common, in video rentals and in

[1] When resale price maintenance was permitted it was used in a wide variety of retail markets, including many lines of clothing (jeans, shoes, socks, underwear, shirts), jewelry, sports equipment, candy, biscuits, automobiles, gasoline, small and large appliances (stereos, shavers, washing machines). See Mathewson and Winter (1998). Estimates of the proportion of retail sales subject to RPM in the United States during the 1950's run from 4 percent to 10 percent (Scherer and Ross 1990: 549; Overstreet 1983: 6). In both the United Kingdom and Canada the practice was even more popular than in the United States: In 1960, some 25 percent of goods and services were subject to RPM in the UK, and in Canada, before the law prohibiting RPM was enacted in 1951, an estimated 20 per cent of goods sold through grocery stores and 60 per cent sold through drugstores were fair-traded (Overstreet: 153, 155).

[2] Vertical price ceilings were per se illegal after *Albrecht v. Herald Co.*, U.S.S.C. 1968, but the Supreme Court reversed the law in *State Oil Co. v. Khan,* U.S.S.C. 96-871 and now vertical price ceilings are legal. In *Albrecht v. Herald*, the plaintiff was a newsvendor.

franchising generally.[3] Our framework is also applicable to many other observed contracts.[4]

This paper contributes to three related literatures. The economics literature on vertical restraints (e.g., Deneckere, Marvel and Peck (1996, 1997), Butz (1997), Mathewson and Winter (1984), Katz (1989), Dana and Spier (2001)) has mainly a positive motivation, in explaining observed contracts. The managment science literature has a prescriptive motivation in analyzing the optimal means of managing inventory in a multi-stage distribution system. Cachon's (2003) survey of supply chain coordination cites more than 150 articles and the area has been active since the survey was written. An antitrust literature addresses the normative issue of whether particular vertical restraints should be allowed (Posner (1981), Easterbrook (1984)). Resale price maintenance, for example, is per se illegal in the U.S. and vertical price ceilings have only recently escaped this status.[5]

We begin by outlining the classic newsvendor inventory model as well as the theory of vertical restraints that we will apply to the inventory problem. We then offer our basic model and four propositions, which in turn identify incentive incompatibilities and characterize resolutions. The characterization of some efficient contracts takes a first-order approach to the underlying principal-agent problem, which requires assumptions of concavity in endogenous agent payoff functions. The payoff functions are not concave in general, however, and the characterization of other resolutions to the problem does not depend on concavity of the entire payoff function.

[3]As Dana and Spier (2001) and Mortimer (2004) discuss, the video rental industry recently switched to a rental contract with distributors. Traditionally, retailers such as Blockbuster had bought videotapes from a distributor for about $65 a copy and kept all rental revenues. Under the new system, videos are purchased by outlets for about $8 dollars each; 10 percent of the revenue goes to Rentrak, which is an information management company that runs the system; the remaining revenue is split between the video retailer and the movie studio. (See www.rentrakonline.com.)

[4]Other contractual clauses are **price discount sharing schemes**, which are contractual clauses that provide a functional link between wholesale prices and retail prices; trade promotions known as **bill-backs** and **count-recounts** (bill-backs refer to schemes by which the costs of downstream promotions are shared by the upstream firm); **flexible-quantity contracts**, and so on.

[5]The research papers that lie closest to our model are Bernstein and Federgruen (2005), Deneckere, Marvel and Peck (1997) and Dana and Spier (2001). Bernstein and Federgruen's model is highly complex and sets aside the possibility of demand spillovers from a stocked-out retailer to other retailers. We suggest that this type of externality is central and can be incorporated in a simple framework. Deneckere, Marvel and Peck offer a fundamental contribution in linking resale price maintenance to inventory management in a model with market power upstream and perfect competition downstream. Our model incorporates market power downstream in the form of a differentiated duopoly. This has the advantage of revealing the sources of coordination failure of simple price-mediated exchange and the roles of a wide range of contracts in resolving the coordination failures.

2 Background

The optimal inventory problem for a single firm: The classic newsvendor inventory problem considers a firm sourcing a good at a constant per unit cost, c, and facing an exogenous price, p. The firm must order an amount y of the good prior to the realization of demand and any product not sold is worth nothing. Denoting the distribution of uncertain demand by G, the firm's expected profit is $p\int_0^y xdG(x) + py[1-G(y)] - cy$. Maximizing this with respect to y yields the fractile solution $1-G(y^*) = \frac{c}{p}$. Equivalently, $G(y^*) = \frac{(p-c)}{p}$. Note that if an upstream seller is providing the product to a downstream retailer at a uniform price w then the retailer's first-order condition substitutes w for c, and the retailer therefore orders too little inventory. The standard double-marginalization or vertical externality distorts the retailer's decision: the retailer ignores the upstream margin $(w-c)$ that accrues with each additional unit of y ordered.

The simple newsvendor problem is concave and the first-order conditions are sufficient as well as necessary for the solution. This is not always true in extensions that (a) endogenize price (e.g., Petruzzi and Dada (1999)) (b) involve competing newsvendors with inventories that spillover from one firm to the other in the event of a stock-out (e.g., Netessine and Rudi (2003)); or (c) involve a vertical structure in which an upstream firm relies on inventory decisions by downstream firms (e.g., Lariviere and Porteus (2001)). Since our model will incorporate all three of these effects, we must be sensitive to the possibility of non-concavities.

The simple analytics of vertical price restraints: The structure that we will consider throughout is a single manufacturer upstream producing a single product at unit cost c and selling to two differentiated retailers downstream, who in turn sell to consumers. While our aim is to understand contracts that coordinate decisions on pricing and inventory, it is helpful to set out in advance the structure in which demand is certain and depends on price and another retailer action such as sales effort or service.[6] Assume that the demands for the product downstream at the two retailers are symmetric, with the demand at retailer 1 given by $q_1(p_1, s_1; p_2, s_2)$ where p_i and s_i are the price and sales effort or service – measured in units of the dollar cost of the effort – provided at the two retailers. Retailers bear no cost other than the wholesale price, w, paid to the upstream producer and a fixed fee paid to the

[6]This framework is developed in more detail in Winter (1993).

producer for the right to carry the product. The profit for retailer 1 gross of the fixed fee is denoted by π_1, and the total profit, for the manufacturers and both retailers is denoted by Π. These profit functions are given by

$$\begin{aligned} \pi_1((p_1, s_1; p_2, s_2) &= q_1(p_1, s_1; p_2, s_2)(p_1 - w) - s_1 && (1) \\ \Pi((p_1, s_1; p_2, s_2) &= q_1(p_1, s_1; p_2, s_2)(p_1 - c) + q_2(p_1, s_1; p_2, s_2)(p_2 - c) - s_1 - s_2 && (2) \end{aligned}$$

Assume that Π is maximized at a symmetric set of prices and effort levels and denote this optimum by (p^*, s^*). Furthermore, for purposes of this background assume that π is concave. Can (p^*, s^*) be achieved with a single instrument, w, or are more complex contracts required? The key to understanding any incentive distortions in retailer decisions is to isolate and decompose the difference between the marginal gain in *individual* profit and the marginal gain in *total* profit from a change in either p_i or s_i. From equations (1) and (2), it follows that at a symmetric configuration ($p_1 = p_2 \equiv p$ and $s_1 = s_2 \equiv s$):

$$\frac{\partial \pi_1}{\partial p_1} = \frac{\partial \Pi}{\partial p_1} - \overbrace{\frac{\partial q_1}{\partial p_1}(w - c)}^{\text{vertical externality}} - \overbrace{\frac{\partial q_2}{\partial p_1}(p - c)}^{\text{horizontal externality}} \tag{3}$$

$$\frac{\partial \pi_1}{\partial s_1} = \frac{\partial \Pi}{\partial s_1} - \overbrace{\frac{\partial q_1}{\partial s_1}(w - c)}^{\text{vertical externality}} - \overbrace{\frac{\partial q_2}{\partial s_1}(p - c)}^{\text{horizontal externality}} \tag{4}$$

and similarly for retailer 2. The individual retailer's private optimum in setting p_1 (where $\partial\pi_1/\partial p_1 = 0$) is distorted from the collective optimum by two externalities: when p_1 is raised, the manufacturer's collects the wholesale markup, $(w - c)$, on a smaller demand through retailer 1. This is the *vertical* externality. The effect of this externality is to distort the retailer's price upwards. The second externality operates through the cross-elasticity of demand between the two retailers. When p_1 is raised, the competing retailer collects the retail markup $(p - w)$ on an additional $\partial q_2/\partial p_1$ units and the manufacturer collects the wholesale markup $(w - c)$ on the same additional units; these add up to the term labelled *horizontal* externality in equation (3). The same two externalities distort the sales effort decision. For each instrument, the vertical and horizontal externalities act to distort the decentralized decision in opposite directions.

When will the right choice of w alone elicit the optimum p^* and s^*? That is, when can simple price-mediated exchange lead to efficient price and effort decisions? This efficiency will hold if the last two terms of equation (4) sum to zero when the last two terms of equation (3) sum to zero. It follows immediately that the following is a necessary and sufficient condition for efficiency of the simple contract. Let $\epsilon_{pi}, \epsilon_{pm}$ represent the price-elasticities of individual outlet demand and market demand evaluated at (p^*, s^*), and similarly for $\epsilon_{si}, \epsilon_{sm}$.

$$\frac{\epsilon_{pi}}{\epsilon_{pm}} = \frac{\epsilon_{si}}{\epsilon_{sm}} \tag{5}$$

The firm elasticities of demand are of course higher than the market elasticities. Efficiency without restraints requires that the two elasticities be higher in exactly the same proportion. If the left hand side of (5) is greater than the right hand side, then the manufacturer faces a *bias in competitive strategy* downstream: the outlets are excessively oriented towards price competition away from service or sales effort competition, relative to the aggregate profit maximum. This condition can be viewed as a direct consequence of the Dorfman-Steiner (1954) Theorem, which implies that the optimum $\frac{s^*}{p^*q^*}$ equals $\frac{\epsilon_{sm}}{\epsilon_{pm}}$ and the outlet will set $\frac{s}{pq}$ equal to $\frac{\epsilon_{si}}{\epsilon_{pi}}$; the outlet's decision then implements (p^*, s^*) only if $\frac{\epsilon_{sm}}{\epsilon_{pm}} = \frac{\epsilon_{si}}{\epsilon_{pi}}$, which is equivalent to (5).

Vertical restraints can be explained simply as a response to this bias. Continuing with the case of excessive price competition (where the left-hand side of (5) is greater), it is easily shown that at w_p^*, where p^* is elicited, the last two terms of equation (3) are negative, with the result that $s < s^*$ is elicited by w_p^*. Suppose, however, that a price floor $\underline{p}$ is established at $\underline{p} = p^*$ and consider the retailer's marginal profitability of s_i (from (1)):

$$\frac{\partial \pi_1}{\partial s_1} = (p^* - w)\frac{\partial q_1}{\partial s_1} - 1$$

Lowering w raises this marginal benefit because the price floor prevents p from following w downwards. Lowering w to the right value elicits s^*. The two instruments, w and $\underline{p}$, are sufficient to elicit the optimal values of the two targets, p^* and s^*. The non-contractibility of effort is circumvented entirely through the design of a vertical restraint contract under which the vertical and horizontal externalities on effort are exactly offsetting. In sum,

Lemma 1 *An upstream firm selling to two downstream retailers that face symmetric demands $q_i(p_1, s_1; p_2, s_2)$ yielding concave payoff functions can achieve a symmetric optimum (p^*, s^*) with two part pricing alone; with two part pricing and a resale price floor; or with two part pricing and a resale price ceiling; depending on whether the left hand side of equation (5) is equal to, less than, or greater than the right hand side.*

Of course, this lemma does not explain *why* downstream outlets should be biased one direction or the other. Why, for example, were price floors observed much more often than price ceilings (during periods when these restraints were both legal)? The externality-balancing argument merely provides a structure, which we apply below to a setting in which the downstream decisions are price and inventory.

3 The Model: Competing Newsvendors in a Vertical Setting

3.1 Assumptions

In this model, a single manufacturer sells the product through two competing retailers and must either rely on the retailers to choose prices and inventory levels or constrain their choices. We assume that contracts can constrain prices but cannot constrain inventory levels.[7] The demand for the product at the two outlets depends on the realization of a random variable, θ, as well as the two prices: $q_1(p_1, p_2, \theta)$ and $q_2(p_1, p_2, \theta)$. The retailers set prices and choose inventory levels, y_1 and y_2, before the realization of uncertainty. We define $\lambda_1(p_1, p_2, \theta)$ as the proportion of any excess demand from outlet 1 that would spill over to outlet 2 in the event of a stock-out at outlet 1, and similarly for λ_2. (This imposes a restriction that λ_1 not depend on y_1.) Finally, we assume that the joint distribution on the random demands is differentiable and symmetric in the sense that the marginal distributions of $q_1(p_1, p_2, \theta)$ and $q_2(p_2, p_1, \theta)$ are identical. For fixed $p \equiv (p_1, p_2)$, the support of the distribution of $q_i(p_1, p_2, \theta)$ is assumed to be an interval.

The following is a structure that is described by this reduced form. Assume that the demands are derived from choices by a set of consumers each of whom purchases at most one unit of the product. A consumer is identified by a pair of reservation prices (r_1, r_2) where r_i is the consumer's gross value of purchasing the product from retailer

[7] In the language of contract theory, this is an "incomplete-contracting approach" (e.g., Bolton and Dewatripont (2005), Chapter 11) because within the informational constraints of the model presented below, contracting on inventory levels would clearly be possible. In the simple model, as in the supply chain incentive literature, the restriction against contracting on inventory is intended to reflect the fact that in the complex environment of real world, there are strong reasons (especially related to retailer private information) for leaving inventory decisions decentralized. We incorporate these reasons explicitly, if briefly, in Section 3 of the paper.

i. The density of tastes is given by a differentiable function $k(r_1, r_2; \theta)$ on $\mathbb{R}^2$ and we represent $K(S, \theta) \equiv \int_S k(r_1, r_2) dr_1 dr_2$ for any (Lebesgue-measurable) set $S \subset \mathbb{R}^2$. Define $S_1(p_1, p_2, \theta) \equiv \{r_1, r_2 | r_1 - p_1 \geq r_2 - p_2 \text{ and } r_1 - p_1 \geq 0\}$ as the set of consumers who would choose to purchase from retailer 1 and similarly for $S_2(p_1, p_2, \theta)$. And define $H_1(p_1, p_2, \theta)$ as the set of consumers who would prefer to purchase from retailer 1 at the given prices but, if retailer 1 is out of stock, would then purchase from retailer 2: $H_1(p_1, p_2, \theta) = \{r_1, r_2 | r_1 - p_1 \geq r_2 - p_2 \geq 0 \ ; \ r_1 - p_1 \geq 0\} \subset S_1(p_1, p_2, \theta)$. Similarly for $H_2(p_1, p_2, \theta)$. Then $\lambda_1(p_1, p_2, \theta) = K(H_1(p_1, p_2, \theta), \theta)/K(S_1(p_1, p_2, \theta), \theta)$. The assumption that λ_1 depends only on (p_1, p_2, θ) and not on y_1 is interpreted as the restriction that should firm 1 stock out, then all consumers in $S_1(p_1, p_2, \theta)$ are *equally likely* to be left with their orders unfilled.[8] Consumers in this model, if they do bear transactions costs from shopping (which is implied by the preference for one retail outlet over another), place their orders or reserve units before leaving home rather than shopping on the basis of expected fill rates.[9]

The timing of the game is as follows:

1. the manufacturer offers contracts, which in the simplest case include w and F, the fixed fee;[10]

2. the two retailers observe both contract offers and simultaneously choose to accept or not;

[8] This rules out, for example, a "sequential purchase" interpretation in which consumers who have a very strong relative preference for purchasing from outlet 1 are likely to rush, arrive at the outlet early, and have their orders filled. It is a natural assumption in a static model.

[9] A natural setting that reduces preferences further, to a single dimension, and which we will explore in a subsequent section of the paper, draws on a standard representation of differentiated-product duopoly: two firms sell an identical physical product and are located at the ends of a unit line segment; consumers are located along the line segment and have a common value for the product as well as a common travel cost; a consumer purchases from the store offering the lowest price net of transportation cost. We add to this model a random distribution of consumers and the requirement that firms choose inventory and prices before the realization of this demand. The realized distribution of consumers may be asymmetric, to allow the possibility that only one outlet stocks out even when the ex ante equilibrium choices are identical. (The probability distribution over the distributions of consumers, however, is symmetric.) The sets $S_i(p_1, p_2, \theta)$ and $H_i(p_1, p_2, \theta)$ are determined by three marginal consumers in this structured model, given (p_1, p_2, θ): the consumer indifferent between purchasing at the two outlets and the consumer indifferent to purchasing at outlet i and not purchasing, for $i = 1, 2$. Figure 1 below depicts this example.

[10] The assumption that fixed fees are feasible is discussed in section 6. Note that in a sales contract under symmetric information, *any* nonlinear pricing function with nonincreasing marginal price (e.g. quantity discounts) can be implemented as a two-part tariff. Our assumption is therefore that nonlinear pricing is possible. Quantity discounts in reality are quite common.

3. the two retailers simultaneously choose price and inventory levels from some intervals $[0, \overline{p}]$ and $[0, \overline{y}]$;

4. uncertain demand is realized;

5. consumers make purchase decisions;

Given decisions $(p_1, y_1; p_2, y_2) \equiv (p, y)$ and demand realization θ, the demand at outlet i can be expressed as $D_i(p, y_j, \theta) = q_i(p, \theta) + \lambda_j(p, \theta)(q_j(p, \theta) - y_j)^+$. The transactions (sales) realized by outlet i are given by

$$t_i(p, y; \theta) = \min\{y_i, D_i(p, y_j, \theta)\} = y_i - (y_i - D_i(p, y_j, \theta))^+ = y_i - O_i(p, y, \theta) \tag{6}$$

where $O_i(p, y, \theta)$ is the number of "overstocks" at outlet i.

The aggregate realized profit of the supply chain can be expressed as

$$\Pi(p, y; \theta) = p_1 t_1(p, y; \theta) - c y_1 + p_2 t_2(p, y; \theta) - c y_2 \tag{7}$$

The realized profit of outlet 1 can be expressed as[11]

$$\pi_1(p, y; \theta) = p_1 t_1(p, y; \theta) - w y_1 - F \tag{8}$$

3.2 Failure of the Price System to Coordinate Incentives

From (7) and (8) we can derive the first-order conditions and compare the individual incentives and collective efficiency in price and inventory:

$$\frac{\partial E\pi_1}{\partial y_1} = \frac{\partial E\Pi}{\partial y_1} - \overbrace{(w - c)}^{\text{vertical externality}} - \overbrace{p_2 \frac{\partial E t_2}{\partial y_1}}^{\text{horizontal externality}} \tag{9}$$

$$\frac{\partial E\pi_1}{\partial p_1} = \frac{\partial E\Pi}{\partial p_1} - \overbrace{p_2 \frac{\partial E t_2}{\partial p_1}}^{\text{horizontal externality}} \tag{10}$$

[11]We suppress the arguments w and F in the function π_i.

The terms labelled in these equations describe the externalities parallel to those outlined in the previous section of the paper. The last equation captures what we think is a fundamental feature of the decentralization of price and inventory decisions in the newsvendor model: controlling for the level of inventory, *pricing decisions are not subject to a vertical externality.* Given the inventory choice of an outlet, the manufacturer has no direct (vertical) interest in the price at which the inventory is resold. The wholesale revenue and profits are completely determined by the inventory purchase. This does not mean that the manufacturer is indifferent as to the pricing decision. To the contrary, the downstream pricing decision is *always* distorted. Because only the horizontal externality is at work, decentralized pricing is biased towards a price that is, at the margin, too low.[12] In sum,

Proposition 1 *In the basic model, unconstrained outlets fail to achieve the efficient outcome* (p^*, y^*).

Note that Proposition 1 requires no concavity assumptions whatsoever – not even assumptions that guarantee the existence of a pure strategy equilibrium.[13] It relies only on the necessary first-order conditions for the aggregate optimum. Economists often view incentive distortions in terms of externalities, or missing markets. Ironically, the source of the inefficiency in downstream decisions here is a missing externality.

3.3 Contracts that achieve coordination

Given that a simple price contract fails to elicit privately efficient price and inventory decisions, which contracts *are* efficient? A difficulty in differentiated Bertrand models as well as models in which firms choose quantities and prices is that payoff functions are not concave or even quasi-concave. This means that the non-existence of pure strategy equilibria is common in many models of differentiated Bertrand competition (e.g., d'Aspremont, Gabszewicz and Thisse (1979)) and in models in which firms choose both prices and quantities (Friedman (1988)). It is clear in the game that we have set out that quasiconcavity and

[12]Recall that the horizontal externality reflects the decrease in the profits flowing to the rival outlet, and the resulting decrease in the fixed fee that the manufacturer can charge up front to any rival outlet that rationally anticipates the externality.

[13]The proposition is easily extended to the case of an asymmetric efficient outcome.

therefore existence of pure strategy equilibria do not hold in general.[14] A common approach in applications of these models is to restrict strategies or parameters to ranges where the pure strategies do exist.[15] We begin by characterizing contractual resolutions that do not rely on concavity of the entire payoff functions. These contracts, which involve vertically imposed prices, or "dictated prices", rather than just price floors or ceilings, depends on concavity of payoffs in the inventory dimension alone. This requirement is easily met. We are interested, however, in whether price ceilings or floors are optimal because we observe both of these contracts and they are opposite instruments. The prediction of which is optimal is a stronger prediction of the analysis. And from a policy perspective, the law is completely different on each restraint so welfare analysis should distinguish the two. Accordingly, we turn subsequently to cases in which the endogenous payoff functions are assumed to be quasi-concave. We compute ranges, in the more structured model of Section 3.4, in which these payoff functions are quasi-concave.

A similar issue arises with respect to the centralized or collectively efficient price and inventory levels. If the two outlets are perfect substitutes for consumers, then it would always pay the monopolist manufacturer, who is committed to setting prices ex ante, to set different prices at the two outlets.[16] If product differentiation (travel costs) are above a critical level in the retail sector, however, then the symmetry in assumed demand will translate into symmetry of the optimum. We restrict attention to a range of travel costs

[14]Intuitively, when the products offered by the two outlets are very close substitutes, then an outlet may attain one local optimum in price by competing intensively with its rival, and another local optimum by forgoing such competition, charging a near-monopoly price, and relying on the chance of a stock-out at its rival to generate revenue. (The parallel with Varian's 1980 model of sales is clear. In Varian's model, each of two retail firms faces a set of loyal consumers and a set of consumers indifferent between the two firms. Only a mixed strategy exists.) Note that a mixed strategy equilibrium exists because payoff functions are continuous and strategy spaces are compact (Glicksberg (1952)), and that our first proposition did not require a pure strategy equilibrium.

[15]For example, Salop (1979) and all of the address model literature on product differentiation (Eaton and Lipsey 1989) restrict attention to parameters where pure strategy equilibrium exist, although for general parameters such existence is not guaranteed.

[16]The logic is essentially the same as in Dana (2001). Suppose, to give a simple example, that the demand curve of each consumer is identical, with a very high choke price, but that there are 100 consumers with probability 0.95, and 120 consumers with probability 0.05. When there is no differentiation among outlets, then instead of setting one price, the vertically integrated manufacturer would profit from choosing inventory and price at one outlet to equate marginal cost and marginal revenue for the first 100 consumers. At the second outlet, the marginal cost is identical, but the expected marginal revenue is only 5 percent as high. Equating marginal cost with expected marginal revenue at the second outlet yields an inventory much smaller than 20 percent of the first outlet's inventory, as well as a very high price.

in the structured model high enough to deal with the existence problem discussed in the previous paragraph and to achieve symmetry for the optimum. (The former problem tends to be the binding constraint.) In the general framework of this section, our more specific characterization of optimal contracts simply assumes concavity and symmetry.

The contractual clauses that we consider are the following: a linear price w and a fixed fee F as before; a resale price floor; a resale price ceiling; a dictated price; a buy-pack policy in which the outlet has the option to sell back the inventory ex post to the manufacturer for a "buy-back price" b; and a per-unit royalty r.[17] In the following, denote by s^* the probability of a stock-out at either outlet at the first-best (p^*, y^*).[18] Denote the own-price elasticity of expected transactions and the cross-price elasticity of expected transactions, evaluated at (p^*, y^*) by $\epsilon_p^t \equiv d \ln Et_1 / d \ln p_1$ and $\epsilon_{pc}^t \equiv d \ln Et_2 / d \ln p_1$. Note that, as in Proposition 1, the following result does not assume that a pure strategy equilibrium exists in the unrestrained game.[19]

Proposition 2 *The first-best (p^*, y^*) can be achieved with*
(a) a dictated price at p^*, a fixed fee and a linear wholesale price given by

$$w^* = p^* s^*$$

(b) a dictated price at p^*, a linear wholesale price and a buy-back price.
(c) a dictated price at p^*, a linear wholesale price and a per-unit royalty on sales.

When we restrict consideration to the range of parameters where the endogenous payoff functions are quasi-concave (a range that includes sufficiently high t, or differentiation between the products), we can get predictions on the *nature* of the price restraints:

Proposition 3 ***(Quasi-concave payoff functions)****: In the basic model, if the payoff functions π_i are quasi-concave in strategies p_i and y_i then the efficient (p^*, y^*) can be elicited with*
(a) a price floor at p^*, a fixed fee, and a wholesale price.
(b) a linear wholesale price w, a fixed fee, and a buy-back price given by

$$b^* = -\frac{\epsilon_{pc}^t}{\epsilon_p^t} p^*$$

[17] We focus first on per-unit royalties and then consider revenue royalties.
[18] That is, $s^* = prob[D_i(p^*, p^*, y^*) \geq y^*]$.
[19] Proofs are in the appendix.

(c) a buy-back price, a linear wholesale price and a resale price <u>*ceiling*</u>*.*

(d) a linear wholesale price, a fixed fee, and a per-unit royalty on sales.

(e) a linear wholesale price, a per-unit royalty and a resale price ceiling.

The role of the price floor in part (a) of the proposition is clear. The missing externality in the outlet's first-order condition on price leaves the outlet with the incentive to drop price below p^*. Setting $w^* = p^* s^*$ then leaves the outlet, which faces a newsvendor problem, with efficient incentives: its optimum is achieved by setting inventory such that the probability of stock out equal to $\frac{w^*}{p^*}$ (from the well-known solution to this problem), and this ratio is selected by the manufacturer to equal s^*. The only way the outlet can achieve a probability of stock out equal to s^* is by choosing the first-best efficient y^*.

Recall that the source of the categorical failure of the simplest price-mediated exchange to elicit correct incentives in our model was a missing externality (equation (10)). An optimal buy-back policy in (b) resolves the incentive problem by *creating* a vertical externality of precisely the right magnitude. This contract achieves the efficient solution purely through the use of price instruments to elicit the right incentives; no restraint on retailer actions is necessary. The key is that the wholesale price, w, is free for use purely as an incentive device because the fixed fee is available to redistribute profits.[20]

Contractual solutions that do not involve a fixed fee (Propositions 2(b), 2(c), 3(c), 3(d) and 3(e)) are particularly interesting for two reasons. First, one does not often observe fixed fees in distribution networks outside of the franchising context, although other forms on nonlinear pricing can achieve the same effect and are common. These propositions show that coordination of decisions can be achieved even with uniform pricing. Second, as we illustrate in Section 4, these propositions are completely robust to asymmetric information about the size of the market.

In Propositions 2(c), 3(d) and 3(e), we considered per-unit royalties on units sold. Revenue royalties (revenue sharing) are more common in reality and have been the topic of a

[20]This focus on the incentive effects of variable prices, as opposed to the rent collection role, contrasts with analysis in the management science literature in which the wholesale price is burdened with both roles. The simplicity and generality of our approach, compared to the management science literature, is exemplified by the theory of the buyback policy. Starting with our model and (a) eliminating one downstream outlet; (b) making price exogenous – thus eliminating three of the four possible externalities in downstream decisions – we arrive at the oft-cited result of Pasternack (1985) that a buyback policy can resolve incentive distortions. In addition, we characterize the necessary price restraint as a price ceiling.

number of papers in the coordination literature (Cachon and Lavirière (2005), Dana and Spier (2001)). Surprisingly, the incentive effects created by output royalties and revenue royalties are quite different. Where output royalties required the addition of a price ceiling to achieve first-best, revenue royalties require the addition of a price floor:

Proposition 4 *The first-best (p^*, y^*) cannot be achieved with w and a revenue royalty alone, but can be achieved with:*
(a) with w, a constant revenue royalty and a dictated price.
(b) with w, a constrant revenue royalty and a price floor, if profits are quasi-concave.

In a model with inventory and pricing decisions, a royalty – of any value – induces an individual outlet price that maximize the outlet's expected revenue, given the chosen inventory level (i.e. a price where the price elasticity of expected transactions is -1). Revenue royalties have no direct impact at all on price. If there were a single outlet downstream, this price would implement the first-best price since the collective objective, given the inventory level, is also the maximization of expected retail revenue at the single outlet. The wholesale price and royalty could then fill the two roles of eliciting the right inventory and re-allocating rents to the upstream firm. But with a second outlet downstream, individual revenue is always maximized at a lower price than the price that maximizes collective revenue because only the horizontal externality is in effect.[21] A price floor is therefore the right instrument to correct the distortion.[22]

3.4 Spatial Model Example

To illustrate the results presented in this section and to explore the welfare implications of price restraints, consider the following spatial model. Assume that the two retail outlets are located at two ends of a unit line segment and the travel cost per unit distance is t. Each

[21] Equation (17) of the proof does reveal an externality in the first two terms. These terms sum to zero, however, at the optimum (p^*, y^*) from the first-order condition determining p^*.

[22] Inventory choice by vertical outlets has an additional horizontal and vertical effect, beyond the externalities incorporated in our model. An increase in inventory will attract customers who value a well-stocked outlet or high fill rate (and who cannot shop from home). (See Dana and Petruzzi (2001).) In a companion paper (Krishnan and Winter (2005)), we examine the consequences of this "ex ante" effect of inventory for vertical control, in a model that also incorporates dynamics. The impact of the ex ante effect depends critically on the availability of the product – a decision set aside in the newsvendor framework of the current paper.

outlet serves a set of "loyal" customers who incur zero travel cost to shop at their preferred outlet and infinite cost of travelling to the other outlet. Each outlet also competes over the "common" customers, or comparison shoppers, who are uniformly distributed on the line segment joining the two retailers.[23]

Each of the comparison shoppers buys either zero or one unit, and all of these customers have the same reservation price r. This generates a linear demand curve for these consumers, with a choke price of r. We assume that the demand curve generated by loyal customers is also linear, and for simplicity that the elasticity of demand for this demand is the same as for the comparison shoppers. That is, the linear loyal customer demand for outlet i also has a choke price of r: $LD_i(p_i, \theta_i) \equiv \theta_i(r - p_i)$. With the density of comparison shoppers given by a random variable θ_c, the distribution of demand is given by the joint distribution $G(\theta)$, where $\theta = (\theta_1, \theta_2, \theta_c)$.

The surplus that a comparison shopper at location x experiences buying from outlet 1 is $r - p_1 - tx$ and from outlet 2 is $r - p_2 - t(d - x)$ and the customer will buy from the outlet where she obtains the higher net surplus. The customers located at $M_1 = \min\{\frac{r-p_1}{t}, d\}$ and $M_2 = \max\{d - \frac{r-p_2}{t}, 0\}$ are the "marginal" customers for outlet 1 and 2 respectively, indifferent to buying at the outlet or not buying the product. If $M_1 > M_2$, there is a non-empty set of customers (in the interval $[M_2, M_1]$) who have positive utility from shopping at both outlets, and customers located at $M_{io} = \frac{M_1+M_2}{2}$ are at the inter-retailer margin; they are indifferent between the two outlets.

All customers first attempt to obtain the product from their preferred (higher surplus) outlet; if their preferred outlet is stocked out, then those customers who are willing to shop at the other outlet will spillover to that outlet. All customers who shop at a particular outlet have the same probability of getting the product whether they buy directly or on rebound from the stocked-out other outlet. That is, we assume "proportional rationing."[24]

[23]Narayanan, Raman, and Singh (2005) consider a similar spatial model but set aside the possibility of asymmetric demand realizations, and therefore spillover demand, at the two outlets. In our model, the existence of "loyal" customers with uncertain demand allows for the possibility of asymmetric demand realizations and demand spillovers even under a symmetric ex ante distribution of demands. Since Varian (1980), models of retail markets incorporating both shoppers and loyal consumers have been common.

[24]To justify consumer behavior in this model, assume that consumers can telephone a retail outlet and "reserve" one unit of the product if it is in stock. Consumers who have a positive surplus at both outlets call their preferred outlet first, and call the second outlet only if their preferred outlet is stocked out. Proportional rationing follows under the assumption that a customer's location in the call sequence at either outlet is uniformly distributed and identical for all potential customers.

The "common" demand at outlet 1 (number of customers on the line segment who will make their first purchase attempt at outlet 1) is given by $CD_1 \equiv \theta_c(\min\{\frac{M_1+M_2}{2}, M_1\})$ and at outlet 2 is $CD_2 \equiv \theta_c(d - \max\{\frac{M_1+M_2}{2}, M_2\})$. The proportion of customers who spillover from outlet i to outlet j is given by $\lambda_{ij} = \frac{(M_1-M_2)^+}{2}/(LD_i + CD_i)$.

Using this set-up, we numerically simulate and compare the centralized and decentralized solutions. Each outlet chooses price p_i and inventory y_i prior to observing demand. In the centralized case a single decision maker chooses (p_1, y_1, p_2, y_2) to maximize collective profits, while in the decentralized case, the outlets independently and simultaneously choose (p_i, y_i).

In comparing the centralized solution with the decentralized equilibrium we restrict attention to symmetric outcomes. For very high travel costs, each outlet is independent of the other (their markets do not overlap), and a unique, symmetric solution will result.[25] For intermediate values of travel cost, the two outlets have overlapping markets and have a symmetric solution. (For sufficiently small travel costs, a symmetric solution does not exist.) Where multiple symmetric equilibria exist, we restrict attention to the equilibrium that maximizes joint profits.

To understand the impact on welfare, consider the use of price floors (as in Proposition 3(a)), applied to the spatial case. In adopting a price floor in this model, the manufacturer trades off a higher retail price and a consequent drop in quantity demanded in exchange for greater inventory and a resulting increase in the expected number of transactions. Does the manufacturer's willingness to make this tradeoff signal an increase in total welfare as well? We numerically verify that centralization can increase total welfare and even consumer surplus alone. A sample of the simulation results is presented in Figure 2. In each of the cases where centralization raises welfare, the centralized outcome can be implemented with a price floor.

4 Asymmetric Information

The problem that we have addressed to this point is, from the perspective of contract theory, incomplete. We have asserted that contracting with a price restraint alone, for example, is simpler than contracting on price and inventory. This is true in reality, and motivates the search for simple contracts. But there is no constraint *within the model* that explains

[25] The uniqueness follows from the specific assumptions on demand that we use in the spatial model.

why contracting on inventory is costly. In the static model, the flow of sales is identical to the stock of inventory – and the manufacturer clearly knows the sales to each retailer.[26] In this section of the paper, we incorporate explicitly one feature of reality that potential limits contracting over inventory or contracts including fixed fees. This is asymmetric information about the size of the market. It is clear that in reality retail outlets or individual franchisees have private information that is relevant to their decision-making. Informational advantages of decentralized agents are key to the theory of decentralized decision-making in general, and information about the size of the local market – and therefore the appropriate quantity of inventory investment – is one of the most important dimensions in which outlets are likely to be better informed.

Accordingly, we consider a model in which uncertainty about demand is realized in two stages. In the first stage, a signal about final demand is realized and observed only by the outlets. This signal is about the size of the overall market, i.e. conditional upon the signal the distribution of final demand remains symmetric. The manufacturer offers a contract to the outlets based on the instruments considered to this point. Specifically, let demand at the two outlets be given by $\alpha q_i(p, \theta), i = 1, 2$ where α and θ are both random variables that are realized in sequence. After the realization of α, which is observed by the outlets but not by the manufacturer, the manufacturer offers a take-it-or-leave-it contract which the outlets accept or reject.

The following proposition is a direct implication of Proposition 3, and the homogeneity of degree 1 of all payoff functions in α.

Proposition 5 *If the payoff functions π_i (conditional upon α) are quasi-concave in strategies p_i and y_i then in the asymmetric information game the efficient (p^*, y^*) can be elicited with*
(a) a buy-back price, a linear wholesale price and a resale price ceiling.
(b) a linear wholesale price, a per-unit royalty and a resale price ceiling.

5 Conclusion

An organization faces an incentive problem when an agent within the organization does not appropriate the full net benefits to the organization of the agent's decisions. The

[26] This contrasts with the model of price and sales effort in the background section of the paper, in which it is reasonable to suppose that effort is not contractible.

organization can respond to an incentive problem by altering the net benefits through internal prices that internalize the nonappropriated returns. Or it can constrain the agent's decisions in some dimensions and ensure that the externalities are internalized in the unrestrained dimensions.

We apply these simple principles in this paper to the coordination of price and inventory decisions in a distribution system. The strategies take the form of reward parameters (the wholesale price, royalties, and buy-backs) and vertical restraints on downstream prices. The incentive problem that leads to the failure of the price system and to the need for more complex contracts we characterize, ironically, as a missing externality.

Price floors and price ceilings are used to address the opposite types of incentive problems. Our starting lemma showed that the need for restraints - price floors, for example - hinges not on whether outlets priced too low, but on whether the optimal *mix* of instruments or decisions at the retail level mirrored the efficient mix. Price floors are optimal in the newsvendor model to counter the missing externality. Contractual parameters apart from price restraints aim to internalize externalities in downstream outlets' decisions. The two instruments, a buyback price and a wholesale price, elicit the efficient pair of targets, p^* and y^*, without the need for any restraints at all, when these instruments are set at levels that create exactly offsetting externalities on price and inventory decisions. The most direct role of the buy-back price is to correct for the missing-externality distortion in the *pricing* decision - contrary to what one might expect, in that buybacks seems directed towards inventory decisions. Royalties play a parallel role, but revenue royalties are unable to correct price distortions. We characterize a subset of the efficient contracts for which the optimal parameters are independent of the size of the market and which are therefore robust to asymmetries in information about the size of the market. The model is the simplest framework possible within which the full range of vertical and horizontal interactions of price and inventory decisions are manifest, and yet it yields a rich characterization of contractual resolutions to incentive problems arising in the distribution of products.

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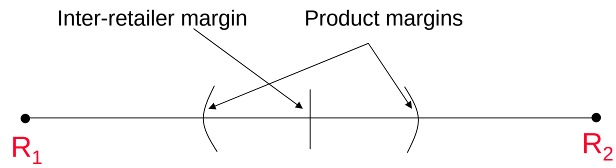

Figure 1: Spatial Model for Section 3

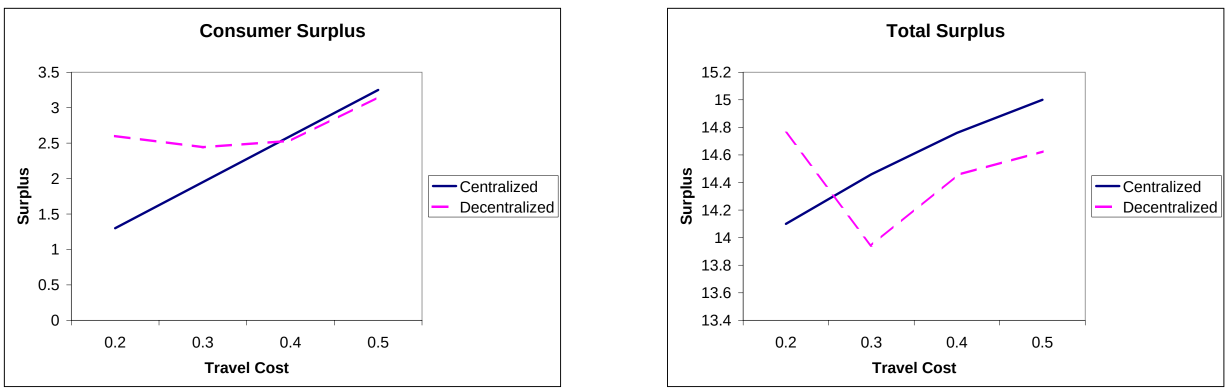

Figure 2: Welfare calculations for spatial model with $r = 2$, $d = 2$, $c = 0.1$, $\xi_1, \xi_2 \sim Uniform\{1, 2\}$, $\xi_c \sim Uniform\{3, 4\}$, t varying

Appendix

Proposition 2: Proof. When the outlets are constrained to charge p^*, then they play a game in inventory decisions y_1 and y_2, with payoff functions are given by $\pi_i(p^*, y, \theta) = p^* \min\{y_i, q_i(p^*, \theta) + \lambda_j(p^*, \theta)[q_j(p^*, \theta) - y_j]^+\} - wy_i - F$. Note that $\pi_i(p^*, y, \theta)$ is concave in y_i and therefore so is $E\pi_i(p^*, y, \theta)$. This guarantees a pure strategy equilibrium in (y_1, y_2). The assumption that the support of $q_i(p^*, \theta)$ is an interval of the real line can be used to show that q_i the payoffs are differentiable. Let $\overline{y}$ be the maximum possible $q_i(p, \theta)$ at (p^*, p^*). Within the range $[0, \overline{y}]$, the reaction function of each firm is differentiable, and has a strictly negative slope.[27] Note also that the optimal response to 0 is positive and that the optimal response to $\overline{y}$ is less that $\overline{y}$. It follows that a *unique* symmetric pure strategy exists.[28]

To prove (a), note that the optimal inventory y^* is characterized by the first-order condition (following from (7))

$$\left[p^* \frac{\partial Et_1}{\partial y_1} - c + p^* \frac{\partial Et_2}{\partial y_1}\right] |_{(p^*,y^*)} = 0$$

From this equation, setting $w = p^* \frac{\partial Et_1}{\partial y_1} |_{(p^*,y^*)}$ ensures that

$$w - c + p^* \frac{\partial Et_2}{\partial y_1} |_{(p^*,y^*)} = 0$$

which then implies that the last two terms of (9) sum to zero, ensuring that the individual and collective first-order conditions for y coincide. Note that $\frac{\partial Et_1}{\partial y_1} |_{(p^*,y^*)}$, proving (a).[29]

To prove (b), note that the outlet 1's realized profit function with a buy-back price is given by

$$\pi_1^b(p, y; \theta) = p_1 t_1(p, y; \theta) + bO_1(p, y; \theta) - wy_1 - F$$

[27] This can be shown by taking the derivative of $E\pi_i$ w.r.t. y_j. The derivative and the expectation operation can be switched by the dominated convergence theorem. We can then differentiate (piecewise) π_i. There will be a range of values of θ where $\frac{\partial \pi_i}{\partial y_j}$ is negative, and a range where it is 0. Taking expectations will give us a strictly negative value - assuming that $\frac{\partial \pi_i}{\partial y_j} < 0$ on a set of measure greater than 0.

[28] A similar argument shows that reaction curves are downward-sloping for the buy-back payoff functions below.

[29] Note that $Et_1 = y_1 - \int_{\{\theta | y_1 > D_1(p,y_2;\theta)\}} (y_1 - D_1(p, y_2; \theta))dG(\theta)$ and $D_1(p, y_2; \theta)$ is a random variable; which we can denote by $\tilde{D}_1$. Define $\tilde{F}(x)$ to be the distribution of $\tilde{D}_1$, i.e, $\tilde{F}(x) = Prob(\tilde{D}_1 \leq x)$. Applying Leibnitz Rule, we get that $\frac{\partial Et_1}{\partial y_1} = 1 - \int_0^{y_1} d\tilde{F}(\tilde{D}_1) = 1 - \tilde{F}(y_1)$, which is the probability of a stockout at outlet 1.

Note that

$$\begin{aligned}\pi_1^b(p,y;\theta) &= \pi_1(p,y;\theta)+b(y_1-t_1(p,y;\theta))\\ E\pi_1^b(p,y;\theta) &= E\pi_1(p,y;\theta)+b(y_1-Et_1(p,y;\theta))\end{aligned}$$

The now-familiar comparison of individual versus collective incentives at (p^*,y^*), with the contractual parameter b, is given by the following.

$$\frac{\partial E\pi_1^b}{\partial y_1}=\frac{\partial E\Pi}{\partial y_1}-\overbrace{\left[(w-c)-b(1-\frac{\partial Et_1}{\partial y_1})\right]}^{\text{vertical externality}}-\overbrace{p_2\frac{\partial Et_2}{\partial y_1}}^{\text{horizontal externality}} \tag{11}$$

$$\frac{\partial E\pi_1^b}{\partial p_1}=\frac{\partial E\Pi}{\partial p_1}-\overbrace{b\frac{\partial Et_1}{\partial p_1}}^{\text{vertical externality}}-\overbrace{p_2\frac{\partial Et_2}{\partial p_1}}^{\text{horizontal externality}} \tag{12}$$

Note that the collective profit function (7) is unchanged with the addition of the buy-back price to the set of instruments, because the buy-back is pure transfer. Since price is constrained, (11) but not (12) is relevant. The task is to find values for w and b that (1) render the sums of the last two terms in (11) equal to zero *and* (2) ensure that the downstream outlets earn zero profits. The first of these conditions ensures that aggregate profits are maximized and the second ensures that all profits accrue to the manufacturer.

This task is met if, when the outlets are constrained by the manufacturer to charge p^*, there is a pair $(\widehat{w},\widehat{b})$ that solves

$$\left[w-c-b(1-\frac{\partial Et_1}{\partial y_1})-p^*\frac{\partial Et_2}{\partial y_1}\right]|_{(p^*,y^*)}=0 \tag{13}$$

and

$$E\pi_1^b(p^*,y^*;\theta)=p^*Et_1(p^*,y^*;\theta)+b(y_1-Et_1(p^*,y^*;\theta))-wy^*=0 \tag{14}$$

The system of equations (13) and (14) is linear in b and w, and linearly independent in these variables, implying that a solution $(\widehat{w},\widehat{b})$ exists.

To prove part (c), note that a contract with a linear price w and a royalty r per unit sold, is pay-off equivalent to a contract with a linear price $w+r$ and a buy-back price r. In both cases, a downstream outlet pays w for all units not sold and $w+r$ for units sold. (A fixed fee need not be part of each contract.) Part (c) then follow directly from part (b) of the proposition. ■

Proposition 3: Proof. As long as one can show that the price floor is binding, then part (a) follows from Proposition 2(a). To prove that a price floor is indeed binding, note that equation (10) shows that $\frac{\partial E\pi_i}{\partial p_i}\Big|_{p_i=p^*} < 0$ and under the concavity assumption on π_i the price floor is binding; and the first-order condition on inventory is sufficient for the optimum, implying that y^* is elicited.

To prove (b), note that setting $b = -p_2 \frac{\partial Et_2}{\partial p_1} / \frac{\partial Et_1}{\partial p_1}$ makes the last two terms of (12) sum to zero. Multiplying the numerator and denominator of the fraction by p^*/Et^* yields the expression for b^*/p^* in the proposition, where t^* is the transactions function evaluated at the optimum. Setting $w = c + b(1 - \frac{\partial Et_1}{\partial y_1}) - p_2 \frac{\partial Et_2}{\partial y_1}$ makes the last two terms of (11) equal to zero. Since $\partial Et_2/\partial y_1 < 0$, this also shows that $w^* > c$.

To prove (c), we need to show that under $\widehat{b}$ (from Proposition 2(b)) the right hand side of (12) is positive evaluated at (p^*, y^*), i.e.,

$$\left[\frac{\partial E\Pi}{\partial p_1} - \widehat{b}\frac{\partial Et_1}{\partial p_1} - p_2\frac{\partial Et_2}{\partial p_1}\right] |_{(p^*,y^*)} > 0 \tag{15}$$

To show (15), note that (12) is zero evaluated at b^* and (p^*, y^*), and that $\partial Et_1/\partial p_1 < 0$; it therefore suffices to show that $\widehat{b} > b^*$. Note that $w^* < p^*$ otherwise an outlet's optimal y is zero. The inequality $w^* < p^*$ implies that $F^* > 0$ in order to meet the zero profit condition under part (b) of Proposition 3. Comparing the zero profit condition (14) with the zero profit condition under part (b), which is identical except for the addition of F^*, we see that $\widehat{b} > b^*$ and/or $\widehat{w} > w^*$. If the former, we are done. If the latter, then setting the last two terms of equation (12) equal to zero (a condition which is satisfied by $b^*$and w^*); and comparing this condition with (13) (which is satisfied by $\widehat{b}$ and $\widehat{w}$), we have from $\widehat{w} > w^*$ that $\widehat{b} > b^*$.

Parts (d) and (e) of the proposition follow directly from parts (b) and (c), because of pay-off equivalence between the buy-back and the per-unit royalty: given any two of the three prices, w, b, u (where u is the per-unit royalty), the third is redundant. ■

Proposition 4: Proof. Letting the royalty rate be μ, the difference between individual and collective incentives can be written as

$$\frac{\partial E\pi_1}{\partial y_1} = \frac{\partial E\Pi}{\partial y_1} - (w-c) - \mu p_1 \frac{\partial Et_1}{\partial y_1} - p_2 \frac{\partial Et_2}{\partial y_1} \tag{16}$$

$$\frac{\partial E\pi_1}{\partial p_1} = \frac{\partial E\Pi}{\partial p_1} - \mu \frac{\partial (p_1 Et_1)}{\partial p_1} - p_2 \frac{\partial Et_2}{\partial p_1} \tag{17}$$

Given any w, at the π_1-maximizing price, $\partial(p_1 Et_1)/\partial p_1$ is zero; hence the middle term of (17) disappears. The last term of (17) is negative, showing that $\partial(E\pi_1)/\partial p_1 < \partial(E\Pi)/\partial p_1$. At a dictated price at p^* or – from the last inequality – a price floor at p^* if profits are quasi-concave, the first-best (p^*, y^*) can therefore be achieved if the externality terms on the right hand side of (16) sum to zero, once μ is set to achieve $\pi_1(p^*, y^*) = 0$. This is achieved simply by setting $w = c - \mu p_1 \frac{\partial Et_1}{\partial y_1} - p_2 \frac{\partial Et_2}{\partial y_1}$. ■