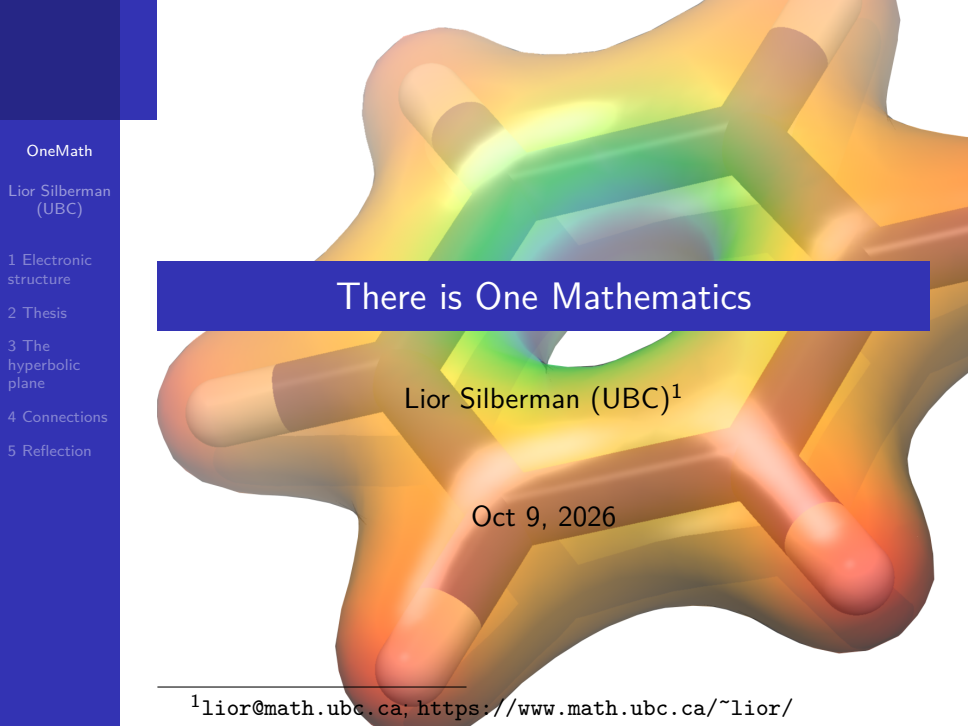




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There is One Mathematics

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Oct 9, 2026

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Plan

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1 Example: Electronic structure

2 The one mathematics

3 Vehicle: the hyperbolic plane

4 Connections:

- a. Probability
- b. Topology
- c. Mathematics physics
- d. $\vdots$

5 Reflection

- a. What have we learned?
- b. What now?

A 6d integral

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The *four-centre overlap integral* with centres $\{\vec{c}_i\}_{i=1}^4 \in \mathbb{R}^3$ and charges $\{\alpha_i\}_{i=1}^4 \in \mathbb{R}_{>0}$ is

$$\iint_{\mathbb{R}^3 \times \mathbb{R}^3} \frac{d^3 \vec{x}_1 d^3 \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|} \exp \left\{ -\alpha_1 |\vec{x}_1 - \vec{c}_1| - \alpha_2 |\vec{x}_1 - \vec{c}_2| \right\} \\ \exp \left\{ -\alpha_3 |\vec{x}_2 - \vec{c}_3| - \alpha_4 |\vec{x}_2 - \vec{c}_4| \right\}$$

- In general use

$$\Phi_i(x) = \frac{1}{N_i} |x - c_i|^{n_i-1} Y_{\ell_i}^{m_i} \left(\widehat{x - c_i} \right) \exp(-\alpha_i |x - c_i|)$$

- Goal: numerically approximate as fast as possible.

Steps

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$$\iint_{\mathbb{R}^3 \times \mathbb{R}^3} \frac{d^3 \vec{x}_1 d^3 \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|} \Phi_1(\vec{x}_1) \Phi_2(\vec{x}_1) \Phi_3(\vec{x}_2) \Phi_4(\vec{x}_2)$$

$$\Phi_i(x) = \frac{1}{N_i} |x - c_i|^{n_i-1} Y_{\ell_i}^{m_i} \left(\widehat{x - c_i} \right) \exp(-\alpha_i |x - c_i|)$$

- (Poisson 1813): $\vec{x}_2 = \vec{x}_1 + r\hat{w}$ with $r > 0$, $\hat{w} \in S^2$.
- (Quake Arena III 1999)

$$\sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2 + (z_1 - z_c)^2}$$

- Why is the grid sum fast?

Alexandra Bünger, William Chen, Mark Ewert, Chen Greif, Avleen Kaur, Gautam Luahana, Ury Segal.

Lessons

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1 Compartmentalize.

2 Connect

- a. Geometry $\leftrightarrow$ integrability;
- b. Order of summation $\leftrightarrow$ cache behaviour;
- c. Number theory $\leftrightarrow$ numerical integration.

3 Meta-compartmentalize: separate knowledge from ideas.

The One Mathematics

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Claim

There is only one mathematics: we all do the same thing.

- 1 All of mathematics is interconnected. We are not that far from each other.
- 2 Mathematics is a *way of thinking*. It can be used in many places and ways, not only internally.

Construction

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(Lobachevsky

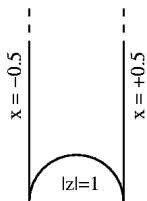
1829, Bolyai 1832; Poincaré 1882)

$\mathbb{H} = \{x + iy : y > 0\}$. For a curve
 $\gamma: [a, b] \rightarrow S$ with $\gamma(t) = (x(t), y(t))$ set

$$L(\gamma) = \int_a^b \frac{\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}}{y(t)} dt$$

and

$$d_{\mathbb{H}}(z, w) = \inf \left\{ L(\gamma) \mid \gamma(a) = z, \gamma(b) = w \right\}$$



- Geodesics: semicircles, vertical lines.
- Sum of angles in any triangle $< \pi$.
- Laws of sines, cosines.
- “Negative curvature”; group theory.

Geometry & Dynamics

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- Midpoints and $CAT(0)$.
- Chaotic dynamics.
- Higher-dimensional generalization: symmetric spaces.

a. Probability

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- Can random groups act on this space?
- How do random walks on this space and others like it behave?

Assaf Naor, David Fisher, (WIP with) Petr Kosenko.

b. Topology

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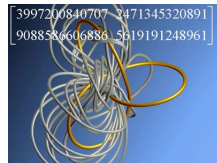
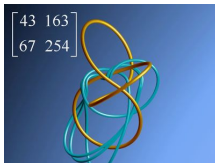
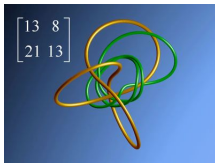
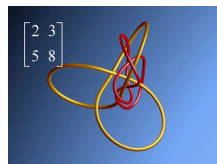
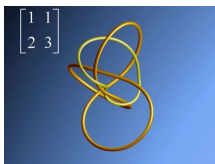
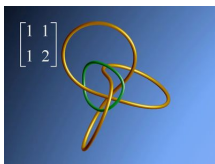
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Theorem (Ghys, Proc. ICM 2006)

Modular knots are the Lorentz knots of Birman–Williams.



Volumes of knot complements

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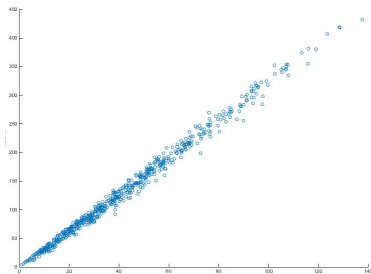
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Theorem (Bergeron–Pinsky–S 2017)

The complement of a modular link of hyperbolic length L has volume $\leq CL$.

■ Numerics: Bergeron–Brands-Longtin–Pinsky–S 2019.



Maxime Bergeron, Alexandre Brandts-Longtin, Tali Pinsky.

c. Mathematical physics

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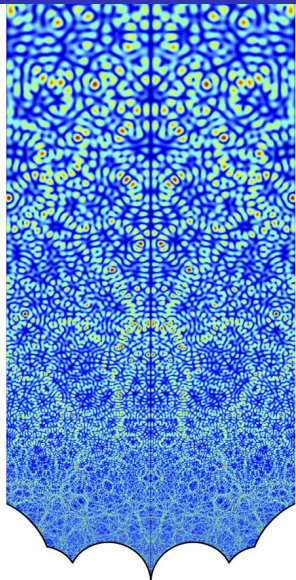
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- *Laplacian* $\Delta = y^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$
- Makes sense on $M = \Gamma(1) \backslash \mathbb{H}$.
- Back to QM: eigenfunctions

$$\Delta \psi = -\lambda \psi$$
$$\|\psi\|_{L^2(M)} = 1$$

- *Symmetries* help here
- Numerical computation (Stromberg)



e. PDE

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Problem

What happens in the limit $\lambda \rightarrow \infty$ (equiv. $\hbar \rightarrow 0$)

- Schnirel'man (1978): connection to the classical (chaotic!) dynamics.
- Schnirel'man–Zelditch–Colin de Verdière: With sufficient chaos, *almost* all ψ_λ become uniformly distributed.

Conjecture (Rudnick–Sarnak 1994)

On a negatively curved manifold, all ψ_λ become uniformly distributed.

- In 2000s progress by Anantharaman et al, Dyatlov et al.

e. PDE

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- Anantharaman: on negatively curved manifolds, limits of ψ_λ must have positive entropy.

Theorem (Anantharaman–S 2013)

Symmetric space version.

- Brooks: non-concentrating *approximate* eigenfunctions.

Theorem (Eswarathasan–S 2017)

Generalization to higher-dimensional hyperbolic manifolds.

f. Representation Theory

g. Number Theory

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- Zelditch 1987: on $\mathbb{H}$ the connection to classical dynamics for hyperbolic surfaces via adapted microlocal calculus.
 - Wolpert 2001: can also be done by explicit spectral theory.
 - Venkatesh–S 2007: Interpretation using representation theory applies on general semisimple groups, with better properties.
-

- Number-theoretic boundary conditions $\Rightarrow$ number-theoretic symmetries of the quantum system
- (Rudnick–Sarnak) $\Rightarrow$ non-concentration of eigenfunctions respecting the symmetries.
- Lindenstrauss 2006: better non-concentration + dynamics gives equidistribution.
- Venkatesh–S, Shem-Tov–S: higher-dimensional version, many cases.

What is mathematics?

“ I believe in the fundamental interconnectedness of all things”²

Definition

Pure mathematics is the branch of philosophy that resolves to only make true statements.

Definition

Mathematics is the branch of philosophy that advances mathematical understanding.

- C.f. Thurston (Bull. AMS 1994)
- Not limited to math. understanding of math!

²Douglas Adams, *Dirk Gently's Holistic Detective Agency*

What now?

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